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FEEDBACK DIFFERENTIAL TRACKING A/D CONVERSION

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Abstract: The result of a measurement has uncertain value-band depending on dynamic behaviour of the measured object and measurement instrumentation. Numerical value is attained with successive approximation of the difference between the reference and measured quantity. Estimation of the difference between two levels is possible with at least two sampling pulses. The dynamic error is proportional to that difference with addition of some constant. For effectiveness of differential tracking, the non-uniform quantization must fulfill three conditions: partitions into halves, increasing quantization uncertainty with difference, and low overlapping of the quantization intervals. The best trade between the number of decision levels and the settling time is with pure exponential quantization rule. The fastest response is achievable with base 2. The conversion time can be shortened to one third in comparison with the best parallel-serial A/D conversions.

Keywords: Differential Tracking, Non-Uniform Quantization, Shorting of Conversion Time

1 INTRODUCTION

Quantization, as part of the A/D (analog-to-digital) conversion, has great accent on the rate-distortion theory [1]. As the chief support for optimization of the rate-distortion trade, the probability density function $f(g)$ of the signal is used [2]. Digitalization consists of prefiltering, sampling, windowing, and quantization. Besides this, the dynamics of measurement A/D channel is also important. Here the trade between the number of references for generating the reference levels and the number of steps of the conversion is presented. Optimal results for the A/D conversion with regard to the time of conversion, resolution, and used references are obtained by the multistep parallel technics [3].

The analysis of digital measurement dynamics [4], [5], [6] and taking account of the real circumstances (finite sampling frequency, finite-time sampling pulses and finite-time sampling interval) show us the time resolution of sampling pulses. They are carrying the amplitude information of the measured signal.

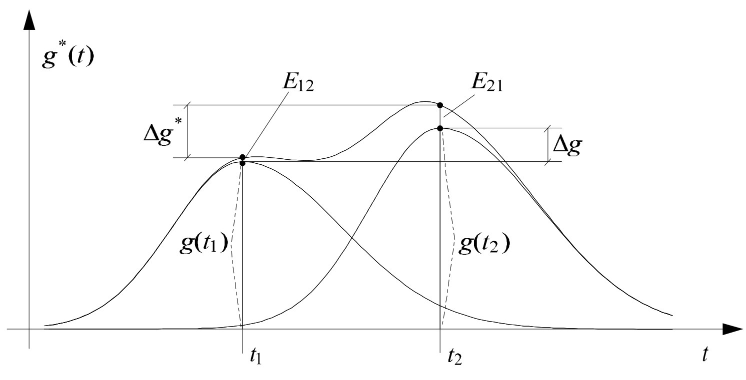

Figure 1. Error of the amplitude difference measurement

During the successive approach to the real value of the measured signal, the distance between the current state and the searched value is estimated at every step. This is possible with at least two sampling pulses (Fig. 1.). With shortening of the sampling time interval to only two pulses we can distinguish between them, when both of them decrease to less than a half of their peak values in the middle time point between the time origins of pulses. When two peaks of the common sampling signal are noted, we distinguish pulses, but the amplitudes of pulses at their sampling origins are not equal to the real values of measured signal, which have to be represented ($g^*(t_1) = g(t_1) + E_{12} = g(t_1) + k_{12}g(t_2)$, $g^*(t_2) = g(t_2) + E_{21} = g(t_2) + k_{21}g(t_1)$). The dynamic error of estimating the difference of two amplitudes is proportional to that difference added up with constant.

$$E_{\Delta g} = \Delta g^* - \Delta g = g^*(t_2) - g^*(t_1) - g(t_2) + g(t_1) = k_{21}g(t_1) - k_{12}g(t_2) = -k_{12}\Delta g + g(t_1)(k_{21} - k_{21}) \quad (1)$$

If sampling pulses have some symmetry in the pulse shape $k_{12} \approx k_{21}$, the error can be written as

$$E_{\Delta g} = k_a \Delta g + k_b \quad (2)$$

2 NON-UNIFORM QUANTIZATION

There are two kinds of dynamic errors: aliasing errors and errors of filtering. Because there are always dynamic errors in the sampling process, it is reasonable to give as the result of measurement the interval of uncertainty, that contains real value, and the representative, that represents that interval and belongs to it. With these intervals it is possible to quantize the measurement range of A/D converter. The optimum spacing of representatives' levels in quantizing intervals is achieved, when they are centres of their intervals or centres of the probability density functions of measured signal appearing in their intervals [2].

Considering the above presumption, it is possible to specify the levels of representatives y_i and the decision levels x_i, which belong to the quantizing intervals Δ_i for the non-uniform exponential quantization.

For effective approach to the real measured value the non-uniform quantization must fulfill three conditions:

- the representative must lie in the middle of the quantizing interval;
- with increased distance between the representative y_i and the current starting-point y_0, also the uncertainty of that representative $\pm\Delta_i/2$ must increase (2);
- quantizing intervals Δ_{y_i} should be crossing each other as low as possible; at the same time they must not leave the empty spaces between them [7].

These conditions give us expressions for decision levels, representatives and quantizing intervals [8]:

$$x_i = (-1)^i \left[2\frac{1-(-a)^{i-1}}{1-(-a)} - \frac{1}{2} \right] \Delta_0 \quad (3)$$

$$y_i = a^{i-1}\Delta_0 \quad (4)$$

$$\Delta_i = \left(\frac{2a^{i-1}(a-1)}{1+a} + (-1)^{i+1}\frac{3-a}{1+a} \right)\Delta_0 \qquad \Delta_0 = \frac{G_{FR}}{2^n - 1} \qquad j = 1, 2, \ldots, n \quad (5)$$

The basis for the pure exponential quantization is limited between 2 and 3 (7).

$$y_i - y_0 \geq y_{i-1} - y_0 \quad \Leftrightarrow \quad \frac{\Delta_i}{2} \geq \frac{\Delta_{i-1}}{2} \quad (6)$$

$$3y_{i-1} \geq y_i \geq 2y_{i-1} - y_{i-2} \qquad 3 \geq \frac{y_i}{y_{i-1}} = \frac{a^{i-1}\Delta_0}{a^{i-2}\Delta_0} = a \quad (7)$$

3 DIFFERENTIAL TRACKING

In the majority of different structures of A/D conversions [9], [10] the numerical value of the measured quantity is attained with reducing the error between the auxiliary reference quantity G_{aur} (Fig. 2.: $G_{aur} \equiv U_p$) and the real value of the measured quantity G ($E_G = G_{aur} - G$). The error is reduced in steps and finally it attains the basic resolution of conversion. It can not be smaller from that which depends on the dynamics of the A/D conversion ($\Delta G_0 = G_{FR} / (2^n - 1)$; G_{FR} - full-scale of the measurement range; n - number of bits).

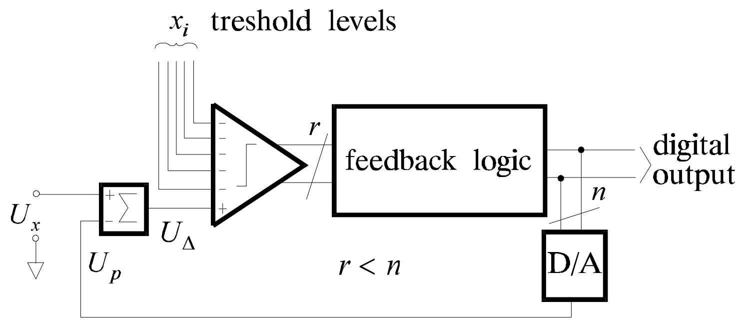

Figure 2. Differential tracking converter

The speed of approximation procedure for different structures of A/D conversions is measured with a number of steps to achieve the basic resolution (Fig. 3.). Each of procedures must fulfill the condition: that the uncertainty $\Delta_j(k)/2$ of previous step $y_j(k)$ is smaller, if compared with the current step $y_i(k+1)$ and added uncertainty $\Delta_i(k+1)/2$ (8).

$$y_i(k+1) + \frac{\Delta_{y_i}(k+1)}{2} \geq \frac{\Delta_{y_j}(k)}{2} \tag{8}$$

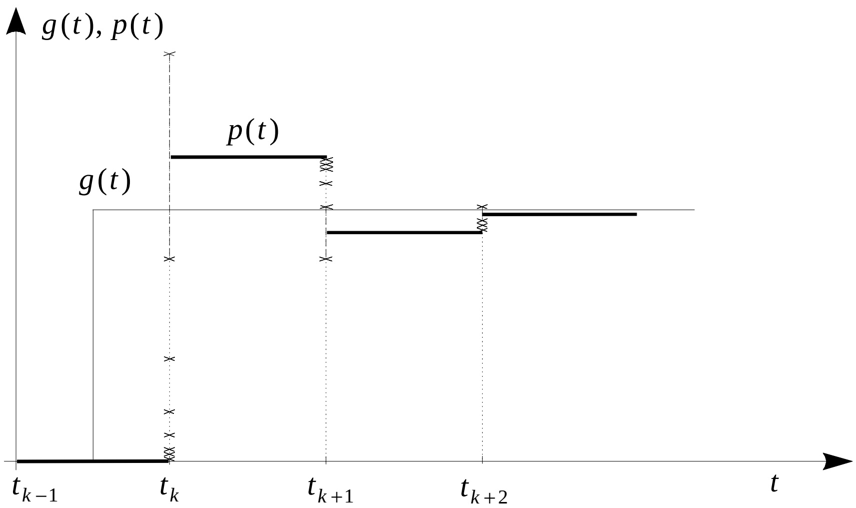

Figure 3. Approximation step response ($a = 2$)

The difference between index $l = j - i$ represents the reduction of current step y_i (9).

$$a^l \leq \frac{2a}{a-1} - (-1)^i \frac{3-a}{2(a-1)} a^{-i+1} \left(1 - (1)^l\right) \tag{9}$$

The fastest response or the shortest settling time in error band of the smallest quantizing interval is achievable with basis $a = 2$, if the basis varies between 2 and 3 (10).

$$l = 1: \quad a \le \frac{2a}{a-1} - (-1)^i \frac{3-a}{a-1} a^{-i+1} \quad \Rightarrow \quad 1 < a \le 3$$

$$l = 2: \quad a^2 \le \frac{2a}{a-1} \quad \Rightarrow \quad 1 \le a \le 2$$

$$l = 3: \quad a^3 \le \frac{2a}{a-1} - (-1)^i \frac{3-a}{a-1} a^{-i+1} \quad \Rightarrow \quad 1 \le a < 1{,}\dot{6}.$$

$$l = 4: \quad a^4 \le \frac{2a}{a-1} \quad \Rightarrow \quad 1 < a \le 1{,}5437 \tag{10}$$

The maximum number of conversion steps for proposed A/D conversion is, at the worst, half of that with the successive-approximation method k_{sa}; the average number is one third.

4 SHORTING OF CONVERSION TIME

The adaptive A/D converter with exponential quantization (basis $a = 2$) can be used in two different ways: with or without sample/hold device. In the first case, the converter approaches with the non-linear proceeding of reducing the uncertainty of the constant measured value. Each conversion is finished with uncertainty of the smallest quantizing interval $\pm\Delta_0/2$. In the comparison with the successive-approximation method, the time of conversion is shortened for two reasons. The difference between approximation and the real value is measured at every step. It is not necessary to begin with test of the most significant bit, if the new value at the input of the converter does not greatly differ from the previous one.

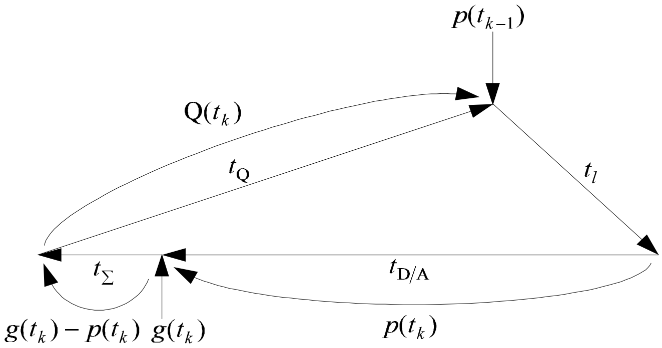

Figure 4. Loop timing diagram of one approximation step

The proposed A/D converter works also without sample/hold device. A part of the sample/hold function takes over the latch-register in the feedback path logic. It holds the value of previous approximation $p(t_{k-1})$, until the estimation $Q(t)$ of difference between the new value of measured signal and the previous approximation is available ($Q = \{y_i\}$).

$$p(t_k) = p(t_{k-1}) + Q(t_k) \tag{11}$$

Within one step (t_{st} - settling time) we have three distinct timing intervals: $t_{D/A}$ - D/A converter settling time and t_Σ - adder response time; t_Q - quantizer response time and t_l - time to accumulate and latch (Fig. 4.). The highest time consumption is $t_{D/A}$ [11]. With the adaptive property of the method, that every previous approximation step to the signal becomes the centre of observation with the exponential increasing resolution in a new step, is possible to shorten the time (12) of a single step (Fig. 5.: $a_{sh} = 0.09, 0.2, 0.5$) and also of the whole conversion.

$$a_{sh} = \frac{t_{sh}}{t_{st}} \tag{12}$$

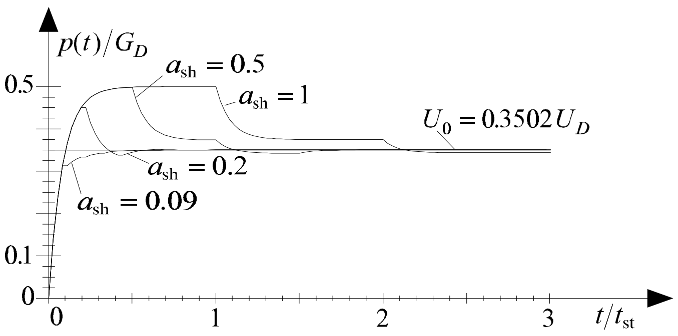

Figure 5. Examples of step cutting for shortening of the settling time ($U_D = G_D = G_{FR}$)

The adaptive property of the method means that every previous approximation step to the signal becomes the centre of observation with exponential increasing resolution in a new step. This property makes it possible to shorten the time of a single step and also of the whole conversion. The conversion time can be shortened to one third ($a_{sh} = 0.15$):

$$t_{a.A/D} = (0.35k + 0{,}45)\, t_{st}(G_{FR}) \quad (13)$$

The central position of representatives causes the smallest possible distortion. Prediction in current step can not give smaller distortion, because the probability density function of the input signal for the next step is unknown. Also with different linear transformations the tracking distortion of the converter can not be reduced, because the method of tracking is non-linear.

In the end we made comparisons between the proposed adaptive A/D conversion ($a = 2$) and the parallel-serial and the successive-approximation conversions according to three parameters: the number of steps k to approaching the basis quantizing uncertainty; the number of standards r for generating decision levels and, commonly, the product $p_{kr} = k \cdot r$ of first two parameters. The adaptive method is equivalent to the optimal parallel-serial method at the worst by maximum possible number of steps $k_{max} \approx n/2$. At the average, the number of steps is smaller than $\bar{k} \approx n/3$. The improvements following the k and p_{kr} parameters are approximately three times at the worst (k_{max}) in comparison with the best parallel-serial A/D conversions (Fig. 6.).

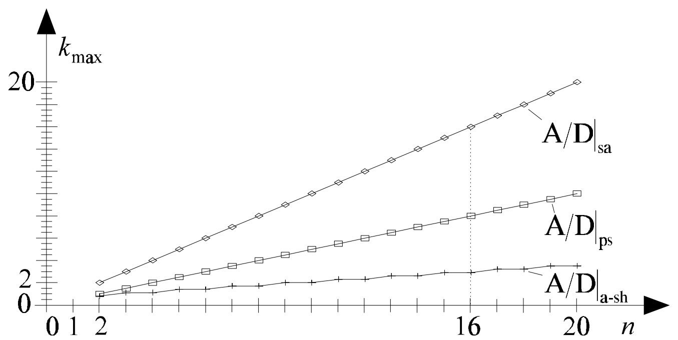

Figure 6. Comparison of parallel-serial ($\mathrm{A/D}|_{ps}$), successive approximation ($\mathrm{A/D}|_{sa}$) and proposed adaptive A/D conversion with step cutting ($\mathrm{A/D}|_{a-sh}$)

5 CONCLUSION

With the proposed principle that the larger as the distance is, the larger is also the dynamic error to measure that distance during the first step, it was possible to develop the adaptive A/D conversion with improvements of the settling time and the measurement-technical consumption. The basis of adaptive operating is currently estimating the difference between approximation and searched value with the non-uniform quantization. The best trade between the number of decision levels and the settling time is with pure exponential quantization rule. The fastest response is achievable with base 2. The adaptive method is equivalent to the optimal parallel-serial method at the worst by maximum possible number of steps $k_{max} \approx n/2$. At the average, the number of steps is smaller than one third.

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