Uncertainty evaluation of totalization of flow and volume measurements in drinking water supply networks

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Abstract

Clean water and sanitation are one of the 17 sustainable development goals (SDG) of the United Nations' 2030 agenda for action, being directly related to several other objectives, namely, economic growth, sustainable cities' communities, responsible consumption and production, and climate action. Since demand for this resource is constantly growing, problems of scarcity of water and transboundary issues are becoming critical to increase water supply efficiency and to improve water management in modern society. Water providers make use of large infrastructures – water supply networks – defined as engineering systems based on hydrological and hydraulic elements able to supply water to consumers, industries, facilities, services and other users. These infrastructures rely on the quality of measurement as a condition to management, having a relevant role in the process of decision-making and to deal with the common problem of water losses. Good measurement practices and uncertainty evaluation are needed to support robust analysis in urban water supply systems. For many water utilities the evaluation of uncertainty is still considered a difficult task, often in situations of missing data for the analysis, having to deal with large amounts of raw and processed data, and requiring support to apply the provisions of the Guide to the expression of uncertainty in measurement (GUM). However, the application of the GUM to the simple mathematical models used in this context makes it possible to obtain simplified equations that can be used in specific conditions of measurement, providing support to non-expert users with more straightforward approaches. Those include measurement of constant flow, totalization of volume at a single measurement point, and sums and differences obtained by combining branches of a network.

1. Introduction

Clean water and sanitation are one of the 17 sustainable development goals (SDG) of the United Nations' 2030 agenda [1], being directly related to several other objectives, namely, economic growth, sustainable cities and communities, responsible consumption and production, and climate action. Since demand for water is continuously growing, the risk of scarcity of water together with non-sustainable water supply and transboundary issues are becoming critical, requiring the increase of water supply efficiency and the improvement of water management in our society [2].

Water utilities make use of extensive infrastructures – water supply networks – defined as engineering systems based on hydrological and hydraulic elements allowing the supply of water to households, industries, facilities, services and other users.

Sustainable management of these infrastructures depends on the use of equipment able to measure many quantities (flow, volume, level, velocity, pressure, temperature, water quality parameters, among others). Measurements are also required to evaluate compliance with conditions established by regulations, technical specifications as well as management requirements of service and trade, becoming a relevant part of governance and of the global economy.

Management decisions are increasingly supported by information provided by measurement [3]. The lack of knowledge regarding measurement data reliability and associated uncertainty is a key issue for the management of water supply networks.

Good measurement practices and uncertainty evaluation are needed to support robust analysis in urban water supply systems, considering its complex distribution structures and the impact of hidden water losses [4-7]. Therefore, a strong motivation for this study is the assumption that the improvement of the quality of measurement will increase confidence in the results, required to support fair trade relations between service providers, clients and consumers.

The common approach in these trade relations is to measure flow or volume of water as the output quantity (e.g., measurement of flow during a time interval, knowing the cross-sectional area of a conduit, allows the volume delivered during the time interval to be calculated).

In a water network, multiple measurement locations allow network flows and water demand to be obtained and defined in a way that enables the inflow and outflow of water to be calculated and the use of this information to evaluate the net balance of the system.

The measurement process includes three stages:

1. Data acquisition of measurements with a certain frequency, generating a time series;
2. Data processing, to obtain the totalized volume for the time interval considered;
3. The combination of totalized volumes for the several locations to evaluate the net balance (sums and differences) of the system.

Frequently, the goal of the process is to have a net balance between water inflow and outflow of a system or subsystem, in some cases subsequently used in trade relations. These totalized results of flow or volume should be conveyed with their associated uncertainties (usually expressed as relative uncertainties), to promote informed decision-making as well as increasing the confidence between the involved agents.

For many water utilities, the evaluation of uncertainty is considered important but still a difficult task, requiring support to apply the provisions of the guide to the expression of uncertainty in measurement (GUM) [8]. Other situations adding complexity include missing data and dealing with large amounts of raw and processed data.

The application of the GUM to simple mathematical models used in this context enables simplified equations to be obtained that can be used in specific conditions of measurement (e.g., measurement of constant flow, totalization of volume at a single measurement point, sums and differences obtained by combining branches of a network), allowing non-expert users to be supported by more straightforward approaches.

2. Measurement of total volume at a single point of a network

There are many technologies and techniques to undertake the measurement of flow and volume, supported by different physical principles (e.g., mechanical, electromechanical, electromagnetic, acoustic, mass, gravimetric) [9].

Flowmeters that are used in flow systems (liquid and gas) are intended to evaluate the rate of a fluid flow (volumetric flow rate), during a time interval (since observations are a time-dependent phenomenon). This equipment is often used because it can provide higher accuracy levels, being able to give indirectly estimates of the volume. Equipment of this kind includes electromagnetic flowmeters of the following types: vortex, swirl, ultrasonic, differential pressure, compact orifice, pitot, variable area and mass Coriolis.

Another type of equipment that can be used is based on the direct measurement of volume during a time interval, usually named as water meters based on electromechanical combinations. Such equipment usually provides less accurate measurements but is practical to use in many industrial and other infrastructures having reasonable accuracy levels for many common applications. Some examples are volume totalizers, oval gear totalizers, oscillating piston totalizers, lobed impeller gas totalizers and turbine totalizers. For such equipment, the measurement output can be interpreted as the integration of the quantity being measured over time.

The definition of volumetric flow rate, Q, is the volume of fluid that passes per unit time (sometimes referred as volume velocity using the symbol $\dot{V}$) and having as SI unit m^3/s. Volumetric flow rate can be obtained using equation (1), which relates the fluid flow velocity, $\boldsymbol{v}$, and the cross sectional vector area, $\boldsymbol{A}$, where measurement takes place:

$$Q = \boldsymbol{v} \cdot \boldsymbol{A}. \qquad (1)$$

Considering a closed conduit with circular geometry having an internal diameter, D, and a cross section orthogonal to the velocity vector ($\theta = 0°$), Equation (1) becomes

$$Q = vA\cos\theta = v\frac{\pi D^2}{4}. \qquad (2)$$

Theoretically, to obtain the total volume from flow rate measurement, an integration over the time interval, Δt, should be made, as in (3):

$$V = \int_{\Delta t} Q \, \mathrm{d}t. \qquad (3)$$

At the experimental level, flow rate measurement is usually obtained at constant time intervals, *Δt*, creating a discrete set of *n* values in a time series. Thus, Equation (3) becomes,

$$V = \sum_{i=1}^{n} (Q_i \Delta t). \qquad (4)$$

If a direct approach is used, n values of volume of fluid, V_i, are obtained using sampling based on a constant time interval (Δt), the estimate of total volume being given by

$$V = \sum_{i=1}^{n} V_i. \qquad (5)$$

The nature of the measuring approach is often related to the water supply process. For the purpose of this study, two types of systems were considered:

- flow with random behaviour (related to users' demands); Fig. 1 typifies the consumption of water measured during a time interval, sometimes allowing to model (predictive) the system demands; and
- constant flow (controlled by the provider or by the user), as illustrated in Fig. 2, during a time interval (e.g. filling a storage tank).

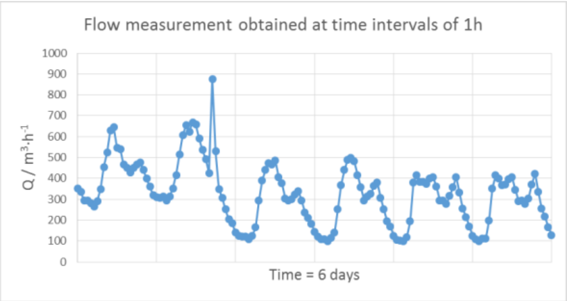

Figure 1: Input flow measurement experimental data obtained in a water distribution network with $\boldsymbol{n}$ users and water losses' during 6 days with time interval of sampling of 1 hour.

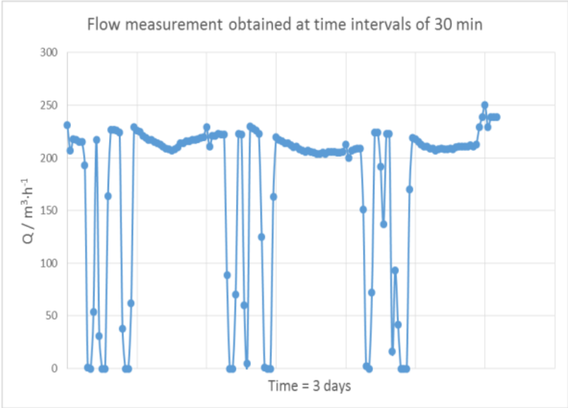

Figure 2: Flow measurement experimental data obtained at the entrance of a storage tank during 3 days with time interval of sampling of 30 minutes.

3. Total volume uncertainty at a single measurement section of a network

Equation (5) provides the functional relation used to obtain the output quantity required, the totalized volume, *V*, given as a sum of discrete values measured during the time of acquisition:

To obtain the standard measurement uncertainty of the totalized volume, the law of propagation of uncertainty (LPU) of the GUM [8] can be applied:

$$u^2(V) = \left(\frac{\partial V}{\partial V_1}\right)^2 \cdot u^2(V_1) + \left(\frac{\partial V}{\partial V_2}\right)^2 \cdot u^2(V_2) + \cdots + \left(\frac{\partial V}{\partial V_n}\right)^2 \cdot u^2(V_n). \qquad (6)$$

Considering that the partial derivatives of V with respect to the V_i are, in this case, all equal to unity, Equation (6) becomes

$$u^2(V) = u^2(V_1) + u^2(V_2) + \cdots + u^2(V_n). \qquad (7)$$

Flow measurement uncertainty is often known and expressed in relative form (proportional to the measured values of the quantity), being its relation with the standard uncertainty, for the case of a single measurement of volume, given by

$$w(V_i) = \frac{u(V_i)}{V_i}. \qquad (8)$$

The use of Equations (7) and (8) yields an equation based on relative uncertainty contributions:

$$u^2(V) = w^2(V_1) \cdot V_1^2 + w^2(V_2) \cdot V_2^2 + \cdots + w^2(V_n) \cdot V_n^2\,. \qquad (9)$$

3.1 Case study of constant relative uncertainty

Measurement of flow in common cases allows to assume that the relative uncertainty is approximately constant for the measurement interval:

$$w(V_1) = w(V_2) = \cdots = w(V_n) = w(V_i), \quad (10)$$

which, when applied to equation (9), gives

$$u^2(V) = w^2(V_i) \cdot [V_1^2 + V_2^2 + \cdots + V_n^2]. \quad (11)$$

This expression simplifies the evaluation of the standard uncertainty of the total volume, as it requires only to know the relative standard uncertainty of the measurement of volume and the *n* values of this quantity. Furthermore, it facilitates the approach used to evaluate the total volume relative uncertainty, again using Equation (8):

$$w(V) = \frac{u(V)}{V}. \quad (12)$$

Consider an example with 10 measurements of volume (experimental data given in Table 1), with relative uncertainty of 2,0 %.

Table 1: Example of 10 measurements of volume obtained using a volumetric counter

V_1	58 m^3	V_6	61 m^3
V_2	63 m^3	V_7	52 m^3
V_3	62 m^3	V_8	57 m^3
V_4	57 m^3	V_9	69 m^3
V_5	79 m^3	V_{10}	76 m^3

With this information, using equations (11) and (12), the total volume standard uncertainty and relative uncertainty, respectively, would be evaluated for the total volume amount of 634 m^3.

$$u(V) = \sqrt{(0{,}02^2) \cdot (40\ 878)}\ \mathrm{m}^3 \approx 4{,}0\ \mathrm{m}^3, \quad (13)$$

$$w(V) \approx 0{,}64\ \%. \quad (14)$$

3.2 Case study of constant relative uncertainty and constant flow

In some particular cases, like the filling of a storage tank, there is a constant flow during an interval of time. In this case, still considering that a constant relative uncertainty is also a condition of the system, we can assume that the relation between measurements obtained at the same interval of acquisition is

$$V_1 = V_2 = \cdots = V_n = V_i. \quad (15)$$

This relation, applied to Equation (11), gives

$$u^2(V) = w^2(V_i) \cdot [n \cdot V_i^2], \quad (16)$$

from which

$$u(V) = \sqrt{n} \cdot w(V_i)\ \cdot V_i, \quad (17)$$

$$w(V) = \frac{\sqrt{n} \cdot w(V_i)\ \cdot V_i}{V}. \quad (18)$$

Equations (17) and (18) are particularly interesting to this study, because they allow the effect of the number of measurement samples (n) in the output quantity uncertainty to be assessed, as detailed in the next section.

3.3 Analysis of the effect of sampling in the constant flow case study

In the design of the process applied to flow measurement, there is not a special concern with the sampling interval, since that interval is often taken according to previous practice or simply adopting the manufacturer's recommendation.

The first rationale for the definition of a sampling interval should naturally be an interval adequate to allow observations of the expected phenomenon variability. However, in special cases like that mentioned with constant flow, intuitively one could think that it had no impact in the level of accuracy of the method.

Looking to Equations (17) and (18), applicable to obtain the standard uncertainty and relative uncertainty of the total volume, respectively, it becomes clear that results depend on the number of samples, n, and on the value obtained of each observation of the volume, V_i. For a certain total volume amount fixed, when n grows, the single observation of volume decreases proportionally.

$$\widehat{V_i} = \frac{V}{n}. \quad (19)$$

Using this relation in Equation (17), a simplified relation is obtained showing that the relative standard uncertainty of the total volume decreases with an increasing number of samples, *n*:

$$w(V) = \frac{w(V_i)}{\sqrt{n}}. \quad (20)$$

Consider a simple example, having a relative standard uncertainty of 2 %, and 10 observations each of 100 m^3, or 5 observations of 200 m^3, with total volume in both cases of 1 000 m^3. Applying Equation (20) the results are, for the data series of 10 values,

$$w(V) = \frac{w(V_i)}{\sqrt{n}} = 0{,}63\ \% \qquad (21)$$

and, for the data series of 5 values,

$$w(V) = \frac{w(V_i)}{\sqrt{n}} = 0{,}89\ \%. \qquad (22)$$

It can be concluded that sampling has a relevant role in the estimated measurement uncertainty achieved, and that it is possible to use simple and direct approaches to obtain the measurement uncertainty from the basic information provided by the acquisition data and known measurement uncertainty associated with the flowmeter.

Although it was expected that a larger number of observations would provide better knowledge about the small variations and better statistics related to the quantity measurement, the application of LPU of the GUM method also shows that the evaluation of uncertainty benefits from this practice, decreasing its value.

4. Volume uncertainty related to net balance at a water supply system network

Most of water supply infrastructures are part of services provided for trade of this resource, in which it is required to make a net balance of inflow and outflow of water volume in the system and in a set period of time. The approach usually taken is based on measuring the volume at different locations and the use of sums and differences to obtain information needed in the economic process. In many cases, it also allows to identify water losses and to evaluate the efficiency of the system.

A functional relation to characterise the net balance is given by

$$V_{\text{net}} = \sum_{i=1}^{n} V_i - \sum_{j=1}^{m} \tilde{V}_j + \delta V_{\text{loss}}, \qquad (23)$$

where V_i represents the n measuring locations of inflow of water into the system, $\tilde{V}_j$ represents the m measuring locations of water outflow of water in the system, and δV_{loss} the water losses during the transfer process.

Applying the LPU to equation (23), noting that the partial derivatives are all equal to one, gives

$$u^2(V_{\text{net}}) = \sum_{i=1}^{n} u^2(V_i) + \sum_{j=1}^{m} u^2(\tilde{V}_j) + u^2(\delta V_{\text{loss}}). \qquad (24)$$

Considering the existence of a similar uncertainty magnitude, $u(V)$, for the inflow and outflow measurement locations, and neglecting the contribution related to the quantity lost, a simplified equation is obtained:

$$u(V_{\text{net}}) \approx \sqrt{(n+m)} \cdot u(V), \qquad (25)$$

which shows that increasing the number of locations used in the net balance will increase the measurement uncertainty of the estimated net volume.

5. Conclusion

Encouraging the sustainable management of water resources is a societal imperative especially in view of the stresses over these limited resources. Adoption of the adequate strategies to increase robustness of the decision-making process supported by measurements includes the adoption of approaches to make measurements increasingly accurate and reliable, applying traceability principles, understanding the information given by uncertainty and promoting good practices.

The focus of this paper is on the evaluation and use of uncertainty for two typical processes observed in water management: measurement of water consumption and filling of storage tanks. In both cases, simple equations were obtained in order to evaluate the output measurement uncertainty associated with totalized volume, considering two stages of analysis, measuring in a single location during a time interval and making the net balance of a network system.

The second stage is particularly relevant, because it is related to a major problem that is known as “hidden losses”. To perform a cause analysis able to identify the share related to measurement and with real losses, measurement uncertainty needs to be considered.

This paper also points out the relevance of an appropriate decision regarding the sampling time interval, an issue often disregarded in practice.

Finally, as expected, the number of locations used in the net balance has impact on the net measurement uncertainty, which can become relevant for large systems having multiple inflows and intermediate connections. This paper quantifies such effects.

Future work in this field includes increasing the knowledge on the impact of these conditions and effects in experimental data from different water service providers. Another topic intended to be developed is related to the influence of sampling time interval in the totalization of volume, namely, the error and uncertainty related to the assumptions of linearity usually considered.

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