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DISCHARGE COEFFICIENTS OF VENTURI TUBES WITH NON-STANDARD CONVERGENT ANGLES

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Abstract: *This paper describes six 100 mm Venturi tubes manufactured in a range of diameter ratios from 0.4 to 0.75. They are standard except for the convergent angles which are either 10.5º or 31.5º. They have been calibrated in water and high-pressure air. It is clear that the data in air from the Venturi tubes with a convergent angle of 10.5º are much smoother than those with the standard or the higher convergent angle. Work on the physical basis of the equation for the discharge coefficient at high Reynolds number is described, and using this work an equation for the discharge coefficient in air has been obtained with an uncertainty of 0.71 per cent. This is much smaller than has been achieved with standard Venturi tubes.*
Keywords: Venturi tubes, Differential pressure meters, Flowmeters

1 INTRODUCTION

There is an increasing desire to use Venturi tubes for wet gas measurement, but to ensure accuracy it is necessary to understand their behaviour in dry gas first. However, the work of Jamieson et al [1] and of van Weers et al [2] and in [3] has shown that their performance in gas is very different from that in water. One way of improving the results in high-pressure gas is to change the design of the Venturi tube. The option of changing the convergent angle is described here.

2 DESCRIPTION OF THE VENTURI TUBES

Six classical Venturi tubes with machined convergent sections were manufactured by Jordan Kent Metering Systems/Seiko for calibration in water and in air with diameter ratio β = 0.4, 0.6 and 0.75 and nominal diameter 100 mm. They were standard Venturi tubes except that three had a convergent angle of 10.5º and three had a convergent angle of 31.5º. They were manufactured to drawings with tight tolerances designed to ensure that where possible the results were not affected by uncontrolled variables. Each Venturi tube was manufactured out of solid metal so that there would be no steps due to welding within the Venturi tube. They were made of stainless steel and were suitable for use at pressures up to 70 bar with ANSI Class 600 flanges. They were designed not only to meet the requirements of ISO 5167-1 [4] but to follow its recommendations. The Standard recommends the use of a divergent angle between 7º and 8º: 7½º was specified for the Venturi tubes used in this project.

So that the results would not be corrupted by the introduction of steps at joins in the pipework, an upstream length of 8*D* and a downstream length of 4*D*, where *D* is the diameter of the entrance cylinder, were manufactured with machined bores; this ensured that in no case was there a step in diameter greater than 0.0035*D* at the upstream flange of the Venturi tube. The lengths of pipework were manufactured by boring out Schedule 80 pipe to the bore of a Schedule 40 pipe. The lengths of pipe and the Venturi tubes were dowelled to ensure concentricity; O rings were used to ensure that there would not be recesses or protruding gaskets. The distance from the upstream pressure tappings to the first upstream flange was 1.1*D*.

In addition to the shorter lengths of pipework already described, an additional 21*D* length was manufactured by welding a 19*D* length of Schedule 40 pipe to a 2*D* length of pipe machined to the bore of the other pipes, smoothing off any step at the weld. This length of pipe was installed with the machined length adjacent to the machined pipe already described, so that there was 10*D* of machined pipework, whose bore matched that of the Venturi tube very accurately, immediately upstream of the Venturi tube. In total there was 29*D* of pipe of the same schedule with no recesses, protruding gaskets or significant steps upstream of the Venturi tube.

The Standard recommends that the radii of curvature at the intersections of the entrance cylinder and the convergent section, the convergent section and the throat, and the throat and the divergent section be equal to zero, although significantly larger values are permitted. The drawings requested a maximum radius of curvature of 1 mm. In order to measure the radius of curvature measurements of profile were made through the convergent and the throat with one trace per Venturi tube. This was only possible for the three Venturi tubes with convergent angles of 31.5º and for the Venturi tube with a convergent angle of 10.5º and diameter ratio 0.75. The average radius of curvature was 5 mm.

The Standard requires that the surface finish of the entrance cylinder, the convergent section and the throat be such that R_a/d shall always be less than 10^{-5}, where R_a is the arithmetical mean deviation of the roughness profile. All the Venturi tubes had $10^{-5} < R_a/d < 10^{-4}$. If the Draft International Standard proposed to replace ISO 5167-1 is accepted the maximum permissible roughness will be increased so that R_a/d shall be less than 10^{-4}.

The pressure tappings were 4 mm in diameter; the throat pressure tappings were of constant diameter for a length of 94 mm and the upstream tappings for a length of 53 mm. Tappings of constant diameter were used because the work described by Jamieson et al suggested that they might be beneficial. The tappings were connected in 'triple-tee' arrangements.

The convergent angles were determined both from three measurements of diameter for each convergent cone (except for the one with an angle of 10.5º and b = 0.4 where these measurements were not made) and from the wall profile measurements from which intersection radii were obtained. The measured value of the convergent angle never differed from the nominal value by more than 0.12º.

The measured throat diameters were within 0.07 per cent of the mean value of the throat diameters at the pressure tappings. The measured diameters of the entrance cylinders were within 0.075 per cent of the mean value of the entrance cylinder diameters at the pressure tappings.

3 CALIBRATION IN WATER

The Venturi tubes were calibrated in water. When fitted against Re_D the slopes of the lines were small: half had a positive sign, although the positive slopes tended to be larger in magnitude than the negative slopes. Over the range of the data the average increase in discharge coefficient with Reynolds number was 0.0013. It seemed appropriate to represent the discharge coefficient, *C*, of each Venturi tube by its mean value: the results are shown in Figure 1. For comparison a line fitted to data from 15 Venturi tubes with standard convergent angles is shown; this is based on work in [3]. As expected the discharge coefficients of the Venturi tubes with a convergent angle of 10.5° are smaller than those with the standard angle owing to the additional loss in the longer throat. The discharge coefficients of the Venturi tubes with a convergent angle of 31.5° are also smaller than those with the standard angle. This may be due to separation at the downstream end of the convergent section.

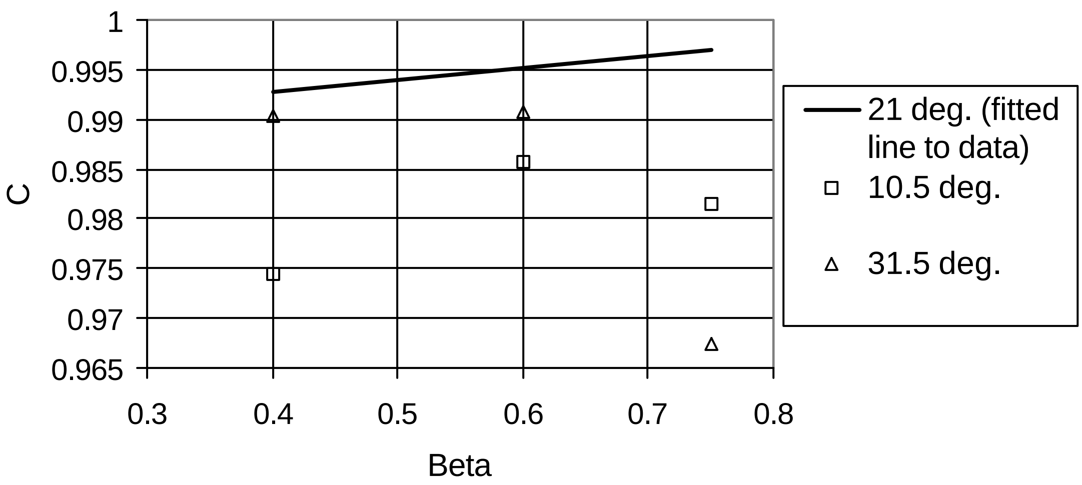

Fig. 1 Discharge coefficients in water

These Venturi tubes together with five with standard convergent angles were calibrated in water both in the NEL Water Bay and in the NEL Multiphase Flow Facility, but of them those with convergent angle

31.5º and diameter ratios 0.6 and 0.75 are the two Venturi tubes which give results significantly different from the pattern of the results taken with the other nine Venturi tubes. They have discharge coefficients which are higher in the Multiphase Flow Facility than in the Water Bay. Perhaps for some reason the flow separated in the throat of the Venturi tube in the Water Bay but not in the Multiphase Flow Facility. Perhaps Venturi tubes with convergent angles significantly larger than the standard value give less repeatable results than ones with the standard convergent angle.

4 CALIBRATION IN AIR

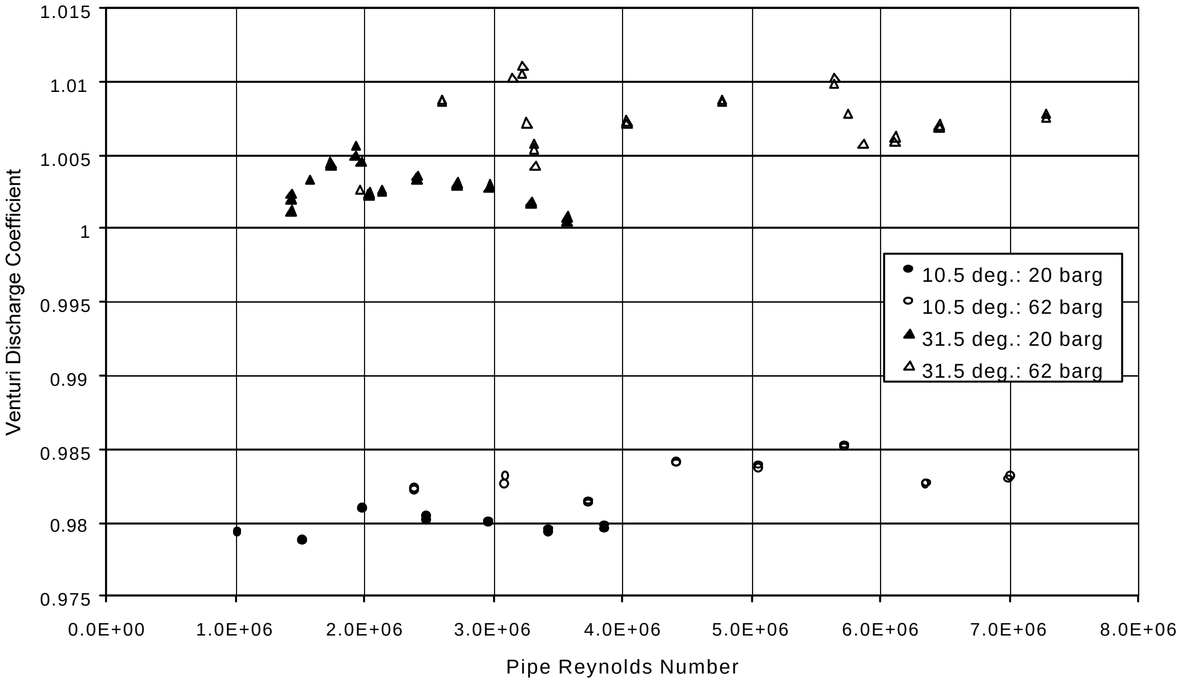

Fig. 2a Calibration in air against Reynolds number: beta = 0.4

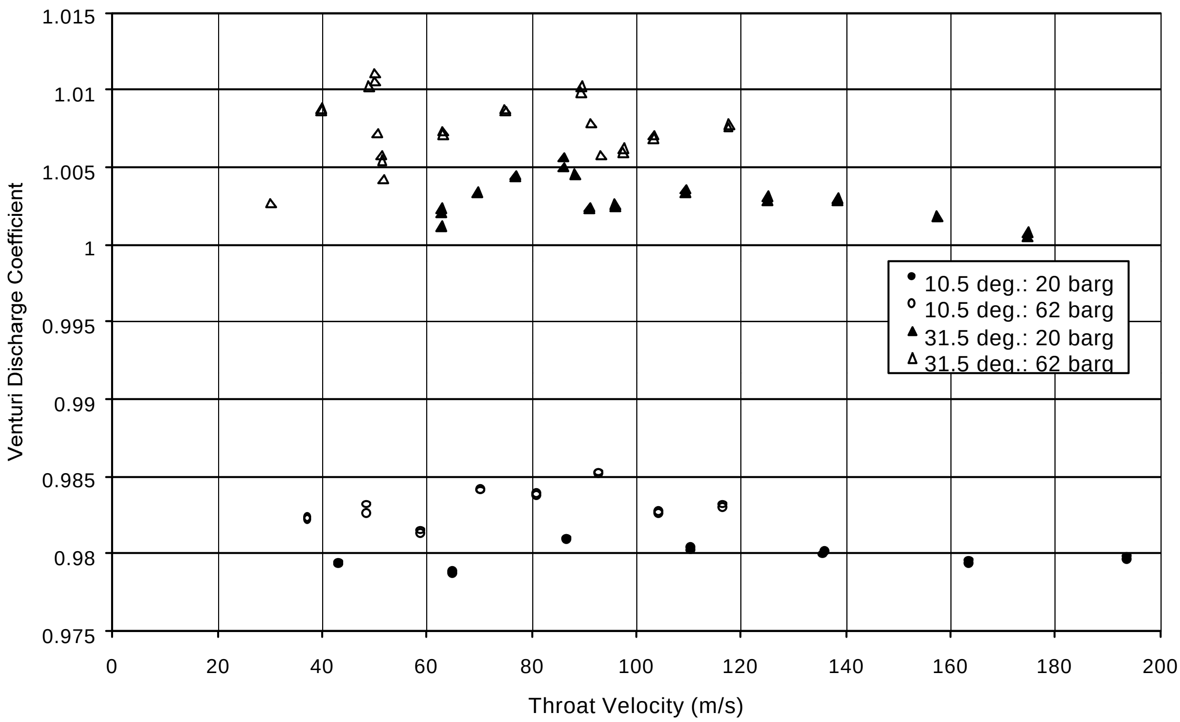

Fig. 2b Calibration in air against throat velocity: beta = 0.4

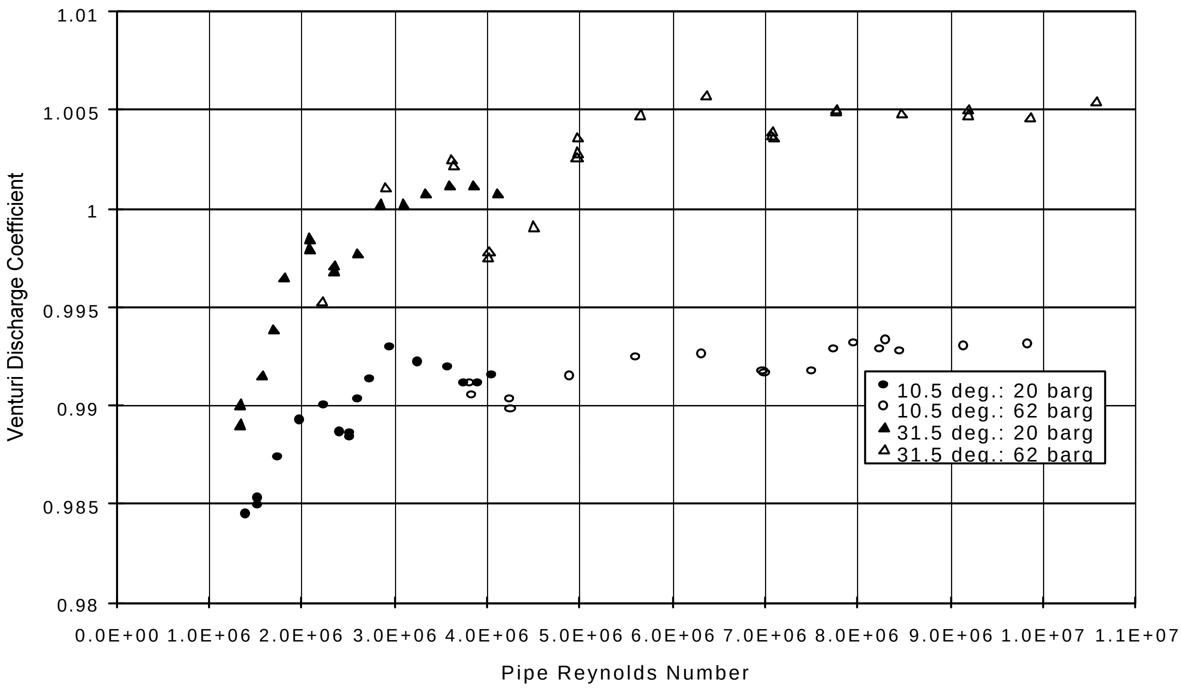

Fig. 3a Calibration in air against Reynolds number: beta = 0.6

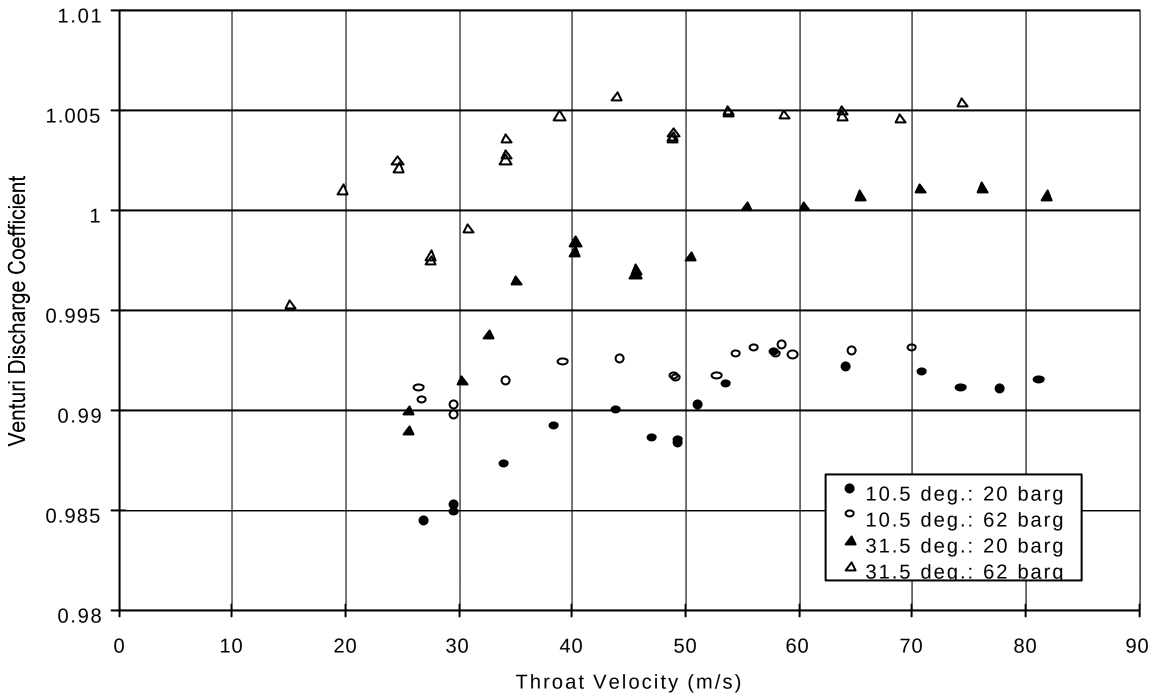

Fig. 3b Calibration in air against throat velocity: beta = 0.6

The Venturi tubes were calibrated in air at two static pressures, 20 bar and 60 bar, and the data are presented in Figures 2 – 4. These data are more scattered than those taken in water. There are peaks and troughs in some of the data sets; the peaks and troughs are larger in the data for a convergent angle of 31.5° than in those for one of 10.5°. In the former case there are throat velocities near which the discharge coefficient changes rapidly. In general better agreement is obtained between the two sets of data obtained at different static pressures when the data are fitted against Reynolds number; however, it is clear that the location of the peaks and troughs is a function of the throat velocity. In all data fitting in this paper the points for which $\Delta p/p_1 > 0.08$ were excluded since these points displayed a reduction in discharge coefficient from what would have been expected from other data from the same Venturi tube. It is assumed that this effect is due to expansibility effects which are not incorporated in the expansibility equation. The difference is, however, less than the predicted uncertainty of ε_1 in ISO 5167-1.

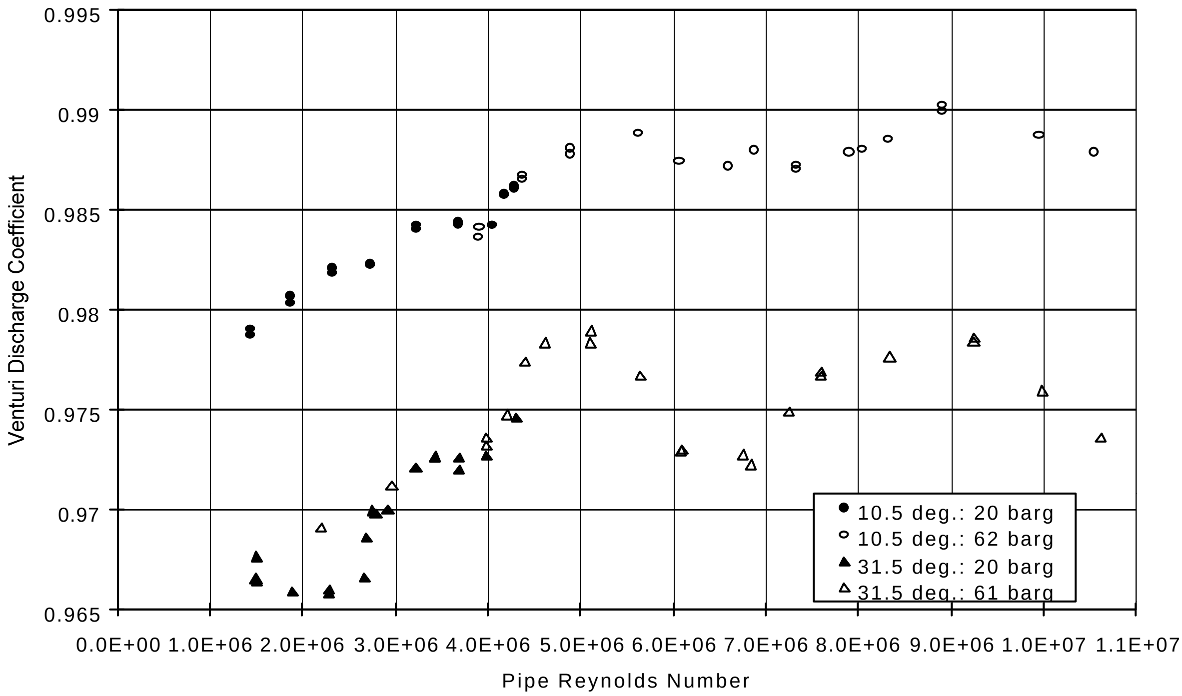

Fig. 4a. Calibration in air against Reynolds number: beta = 0.75

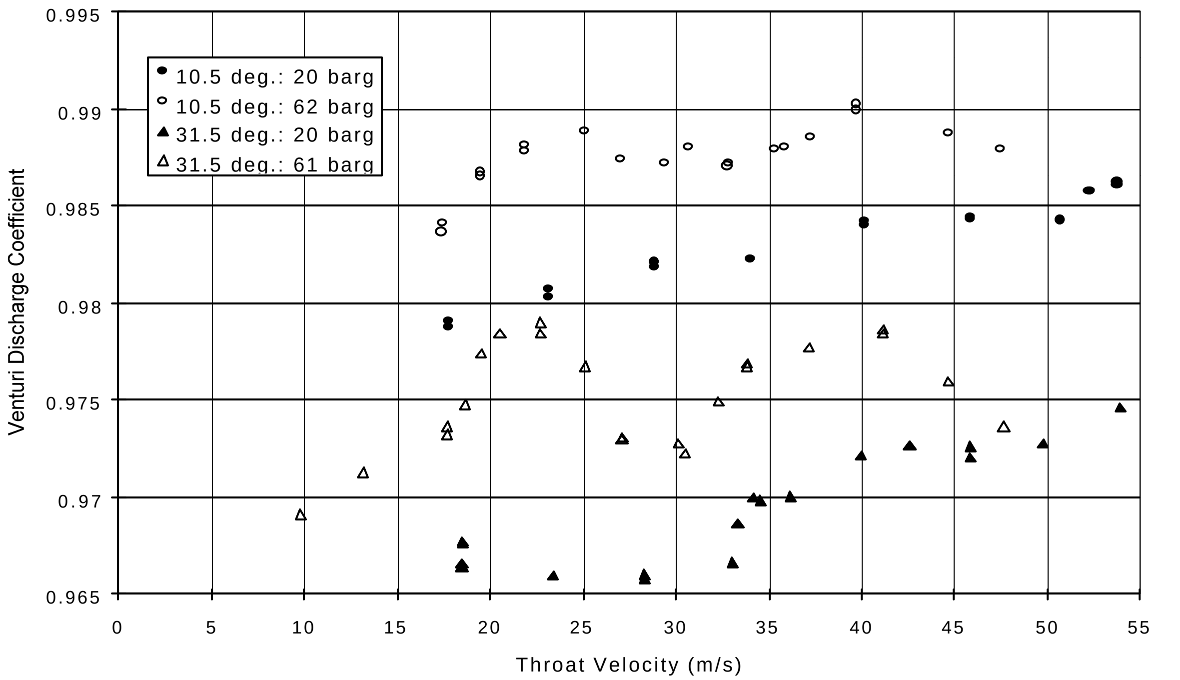

Fig. 4b Calibration in air against throat velocity: beta = 0.75

5 STATIC HOLE ERROR

In order to fit the data it is helpful to observe that some of the variation in C can be removed by examining $C - C_{water}$ where C_{water} is the mean value for the water data for that Venturi tube as shown in Figure 1. A possible cause for the change in discharge coefficient from that obtained in water is static hole error. Static hole error is the effect that pressure tappings of finite size do not measure the pressure which would have been measured using an infinitely small hole. The effect of static hole error is that the measured pressure using a pressure tapping is higher than the static pressure would have been if the tapping had not been present. This effect is considered in many papers (e.g. Franklin and Wallace [5] and Gibson et al. [6]). If the increase in measured pressure is denoted by *e* and the wall shear stress by *t*, then

$$\frac{e}{t} = f\,(Re_{tap})\,, \tag{1}$$

where Re_{tap} is the tapping hole Reynolds number defined by

$$Re_{tap} = \frac{u_t d_{tap}}{n}\,, \tag{2}$$

d_{tap} is the tapping diameter, n is the kinematic viscosity, u_t is the friction velocity, $\sqrt{(t\ /\ r)}$, and r is the density. Because the velocity and therefore the wall shear stress are much higher in the throat than in the entrance cylinder the static hole error leads to a reduction in the measured differential pressure and an increase in the measured value of *C*. To calculate the static hole error it is necessary to have an estimate of the relationship between t and $\overline{u}$, the mean velocity at the tapping plane. Following Schlichting [7] this is expressed in terms of the friction factor, l, where

$$t = \tfrac{1}{8}\, l r \overline{u}^2\,. \tag{3}$$

In a standard Venturi tube in both tapping planes Lindley [8] made measurements of t from which l can be deduced. In the entrance cylinder l appears to be becoming asymptotic to 0.012 as Reynolds number increases. For $10^6 < Re_D < 10^7$ and $k/D = 5 \times 10^{-5}$ l will always be within 10 per cent of 0.012 in a straight pipe according to the Moody Diagram (see Schlichting); so it seems an appropriate value to use. In the throat Lindley's measurements of l are approximately 18 per cent higher than would be obtained in a straight pipe of the same relative roughness and Reynolds number; so in determining the static hole error a figure of 0.015 has been used (for $10^6 < Re_d$ and $k/d = 10^{-4}$, 1.18 l will always be within 6 per cent of 0.015 according to the Moody Diagram).

On this basis (and assuming incompressible flow) the predicted value of the total reduction in differential pressure, e_{total}, is given by

$$e_{total} = \tfrac{1}{8}\overline{u}^2_{throat}\, r(0.015 f\,(Re_{tap,throat}) - 0.012 b^4 f\,(Re_{tap,up})), \tag{4}$$

This corresponds to an increase in discharge coefficient of approximately

$$\frac{0.015 f\,(Re_{tap,throat}) - 0.012 b^4 f\,(Re_{tap,up})}{8(1 - b^4)}\,. \tag{5}$$

In deriving an equation it is necessary to consider the change in *C* from that found in water. Towards the top of a calibration in water a typical value of $Re_{tap,throat}$ is 3000 at which *f* is approximately equal to 3.8; so *f* is written as f^* + 3.8 where $f^*(3000)$ is equal to 0. In water $Re_{tap,up}$ will be less than 3000, but the coefficient of *f* will be larger than 0.012. Assuming that

$$f^* = a(e^{-nRe_{tap}} - e^{-3000\,n}) \quad \text{for } Re_{tap} > 3000 \tag{6}$$

the measured values of $C - C_{water}$ can be fitted, and the best fit of f^* to the data for a convergent angle of 10.5° gives

$$f^* = \begin{cases} 5.229 - 5.896 e^{-0.00004\,Re_{tap}} & \text{for}\, Re_{tap} > 3000 \\ 0 & \text{for}\, Re_{tap} \leq 3000. \end{cases} \tag{7}$$

This fit has an uncertainty based on two standard deviations of 0.0056. The true static hole error (rather than the difference between the static hole error in high-pressure air and that in water) is based on

$$f = 9.029 - 5.896 e^{-0.00004\,Re_{tap}} \quad \text{for } Re_{tap} > 3000 \tag{8}$$

and is shown in Figure 5. When the uncertainty of the complete equation

$$C = 0.9677 + 0.0219\boldsymbol{b} + \frac{0.015 f^*(Re_{tap,throat}) - 0.012\boldsymbol{b}^4 f^*(Re_{tap,up})}{8(1-\boldsymbol{b}^4)} \tag{9}$$

with *f** given by equation (7) is considered, the uncertainty (based on two standard deviations) of the complete database of values of *C* in air is 0.76 per cent.

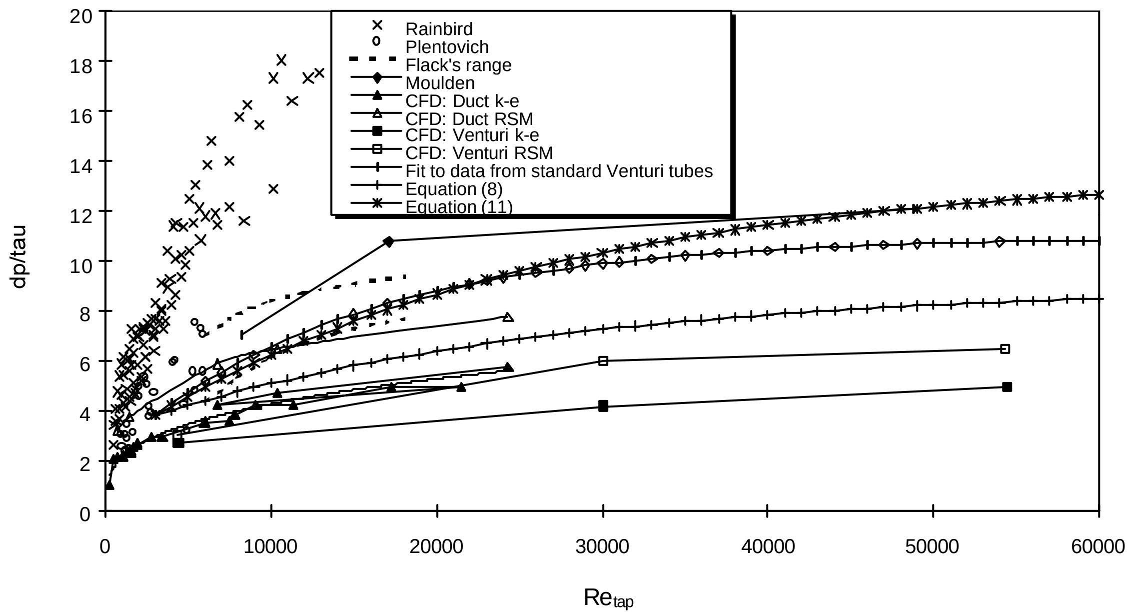

Figure 5 Static hole error

For a convergent angle of 31.5° the best fit of *f** to the data gives

$$f^* = \begin{cases} 9.879 - 11.138 e^{-0.00004\,Re_{tap}} & \text{for}\, Re_{tap} > 3000 \\ 0 & \text{for}\, Re_{tap} \leq 3000. \end{cases} \tag{10}$$

This fit has an uncertainty based on two standard deviations of 0.0130. The true static hole error (rather than the difference between the static hole error in high-pressure air and that in water) is based on

$$f = 13.679 - 11.138 e^{-0.00004\,Re_{tap}} \quad \text{for } Re_{tap} > 3000 \tag{11}$$

and is shown in Figure 5. When the uncertainty of the complete equation

$$C = 1.0189 - 0.0619\boldsymbol{b} + \frac{0.015 f^*(Re_{tap,throat}) - 0.012\boldsymbol{b}^4 f^*(Re_{tap,up})}{8(1-\boldsymbol{b}^4)} \quad (12)$$

with f^* given by equation (10) is considered, the uncertainty (based on two standard deviations) of the complete database of values of C in air is 1.47 per cent.

Figure 5 shows not only Equations (8) and (11) but also an equation based on Venturi tubes with standard convergent angles [3]. None of the equations from Venturi tubes of different convergent angles is inconsistent with the data in Figure 5 (see [6] for the CFD and [9] – [12] for the other data). This is encouraging in that there is a physical explanation for the discharge coefficient values measured in air. Results at high Reynolds numbers, however, depend on other parameters besides Re_{tap}. This can be seen not only in the data shown in this paper but also in the other experimental results, which are very varied at high Reynolds numbers, whereas (see [6]) good agreement between experimental data has been achieved at the Reynolds numbers obtained in water. Moreover, the cause of the peaks and troughs in the data is not clear: they may be the result of unsteady effects (e.g. acoustics), whereas static hole error is a steady effect. These unsteady effects may lead not only to peaks and troughs, but also to an apparent change in the static hole error.

6 PRACTICAL EQUATIONS

An alternative method of presenting the data which is easier to use than the method in Section 5 is to observe that the upstream static hole error term is much smaller than the throat term and that therefore it is possible simply to correlate the data with the throat tapping Reynolds number; the simplest presentation of this is to define the Venturi throat tapping Reynolds number

$$Re^* = \frac{d_{tap}}{d} Re_d \, . \quad (13)$$

The data for C - C_{water} can then be plotted against Re^*, and Figures 6 and 7 give the data for convergent angles of 10.5º and 31.5º.

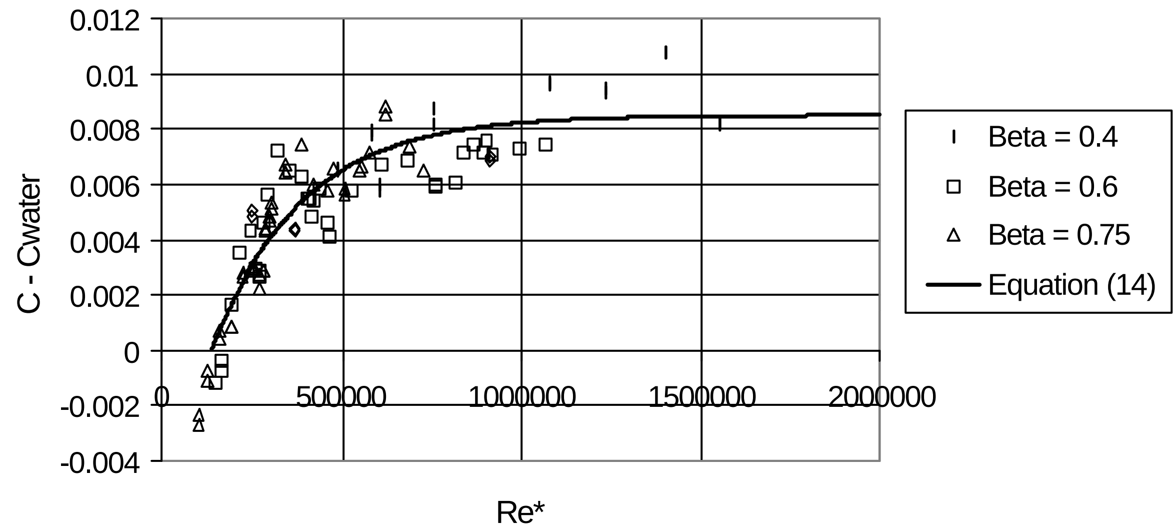

Figure 6 C - Cwater with 10.5 deg. convergent angle

For a convergent angle of 10.5° an appropriate fit for C - C_{water} is

$$C - C_{water} = \begin{cases} 0.0085 - 0.0148e^{-0.4(Re^*/10^5)} & Re^* > 140000 \\ 0 & Re^* \le 140000 \end{cases} \quad (14)$$

This is shown in Figure 6. It has an uncertainty (based on two standard deviations) of 0.0047. When the corresponding overall equation

$$C = \begin{cases} 0.9762 + 0.0219\boldsymbol{b} - 0.0148e^{-0.4(Re^*/10^5)} & Re^* > 140000 \\ 0.9677 + 0.0219\boldsymbol{b} & Re^* \le 140000 \end{cases} \quad (15)$$

is compared with the data for C obtained in air, it has an uncertainty (based on two standard deviations) of 0.71 per cent. This figure is much lower than that obtained for Venturi tubes of standard convergent angle [3]. It is necessary to establish that equation (15) is applicable for general use by undertaking further test work, since it is based on only three Venturi tubes. The standard deviation of the fit to the mean data for a convergent angle of 10.5° in water is 0.42 per cent.

For a convergent angle of 31.5° the fit to the air data is

$$C - C_{water} = \begin{cases} 0.0161 - 0.0281e^{-0.4(Re^*/10^5)} & Re^* > 140000 \\ 0 & Re^* \le 140000 \end{cases} \quad (16)$$

This is shown in Figure 7. It has an uncertainty (based on two standard deviations) of 0.0102. This is more than twice the value for the Venturi tubes with 10.5° convergent angle.

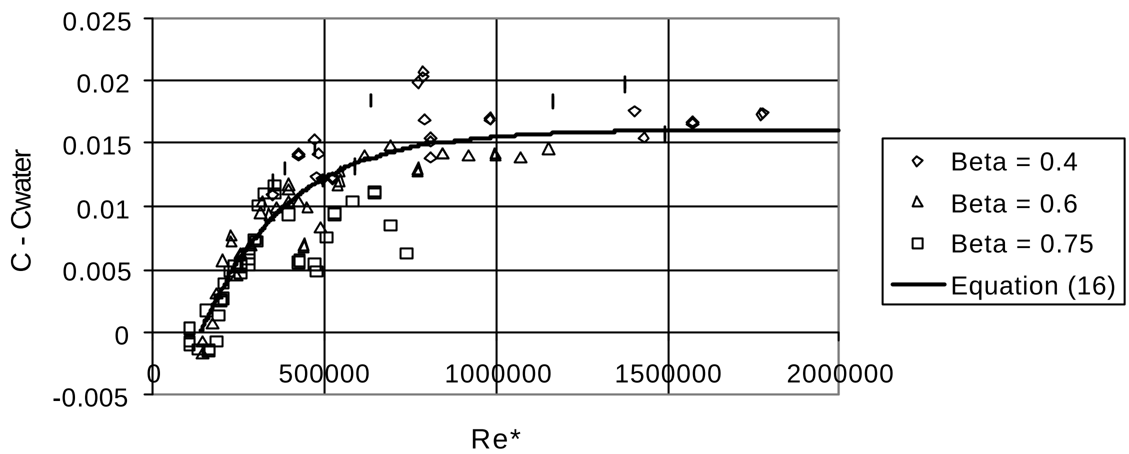

Figure 7 C - Cwater with 31.5 deg. convergent angle

When the corresponding overall equation

$$C = \begin{cases} 1.0350 - 0.0619\boldsymbol{b} - 0.0281e^{-0.4(Re^*/10^5)} & Re^* > 140000 \\ 1.0189 - 0.0619\boldsymbol{b} & Re^* \le 140000 \end{cases} \quad (17)$$

is compared with the data for C obtained in air, it has an uncertainty (based on two standard deviations) of 1.36 per cent. The standard deviation of the fit to the mean data for a convergent angle of 31.5° in water is 0.79 per cent.

7 CONCLUSIONS

Six 100 mm Venturi tubes of a range of diameter ratios have been made and calibrated in water and high-pressure air. The Venturi tubes were standard except for the convergent angles: three had a convergent angle of 10.5º and three had a convergent angle of 31.5º. The results for the three with a convergent angle of 31.5º are less good than for those with the standard angle. The results for the three with a convergent angle of 10.5º are most encouraging; indeed the air data from the three Venturi tubes can be fitted with an uncertainty of 0.71 per cent by an equation which both has a physical basis and gives optimum results in water. The recommended equation is Equation (15) with *Re** defined in Equation (13). The physical basis of this equation is static hole error theory. This provides at least a partial explanation of the measured discharge coefficients. Given the problems with the use of Venturi tubes in high-pressure gas found in earlier work, this uncertainty is very good.

This equation for Venturi tubes with a convergent angle of 10.5° is only based on three Venturi tubes; so further work in this area is desirable.

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