# A NEW DIGITAL SIGNAL PROCESSING METHOD FOR ACCURATE PHASE NOISE MEASUREMENT

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**Abstract –** *The enhancement of a previous digital signal processing method, already proposed by the authors for phase noise measurement, is presented in this paper. The method shows itself particularly suitable for evidencing "close-to-the-carrier" phase-noise, which requires very high frequency resolution. Moreover, its input range results to be much wider than that characterizing the previous method, thus allowing a direct analysis of very high frequency carriers as well.*
*The paper begins with the presentation of the key idea underlying the method; then, it describes in detail the proposed measurement procedure; finally, it points up the results obtained in some experimental tests in order to show the new method perspective.*

**Keywords –** Phase-noise measurement, signal source characterisation, undersampling, aliasing.

## 1. INTRODUCTION

Phase noise of signal sources is a major concern in many application fields, such as communication systems, echo radar detection, satellite positioning, and space telemetry systems [1] [2]. As an example, sideband phase noise is a limiting factor in modern telecommunications systems because it is an inherent cause of interference into the information bandwidth.

Measuring phase-noise generally imposes the evaluation of its base-band power spectral density function. Several systems are available to fulfil this task. They are mainly characterised in terms of measurement principle, input frequency range, and noise-floor.

These systems (such as those based on "Direct Spectrum", "FM Delay Line Discriminator" and "Reference Source/PLL" measurement techniques) compare the source under test to a more stable or at least equally stable source, considered as reference. The main drawback presented by all of them is that they are not capable of granting a low noise-floor, especially for *close-to-the-carrier* phase-noise measurement. This is due to noise contribution of the phase-detector and the internal source, used as reference, to the overall system noise-floor [3].

As an alternative, methods using time-interval-analysers (TIA), which measure carrier zero-crossing fluctuations and convert them to phase units, have also been proposed [4]. Their performance strongly depends on the time resolution, *DT* (20-50ps), of TIA, which introduces quantisation noise. Moreover, they are not suitable to analyse high frequency carriers, because the higher the frequency, $f_0$, of the carrier under test, the worse the phase resolution, *DF* ($DF = 2\pi f_0 DT$).

Recently, the authors have proposed a digital signal-processing method [5]. It requires the digitisation of the RF input sinusoidal carrier by means of a fast converter. Then, in order to evaluate the time-domain evolution of the phase noise, a quadrature demodulation scheme, roughly sketched in figure 1, is adopted. Thus, applying FFT algorithms on the demodulated signal attains the base-band power spectral density function of the phase noise. Furthermore, to achieve satisfying performance, the method claims for a high-resolution analogue/digital converter (ADC), which has to be driven by a stable clock. It is so possible to reduce the contribution both of the quantisation noise introduced by the ADC and the instability of the clock to the overall noise-floor. Unfortunately, high-resolution digitisation can be accomplished only in the presence of low-frequency carriers, which do not require high sample rates. So the method can assure low noise-floor only in the analysis of low-frequency carriers. In perspective, even though the ADC modern technology is reaching higher and higher sample rates as well as analogue bandwidths, a jitter-free high-resolution converter capable of digitising multi-gigahertz carriers in real time is likely to remain part of science fiction for years to come.

Moreover, to achieve a satisfying frequency resolution in estimating the power spectral density of the phase-noise, a long-time-interval capture is needed, and, as a consequence a large memory depth. For example, a memory depth of 1 Mbyte allows a time interval capture of 1 ms if a sample

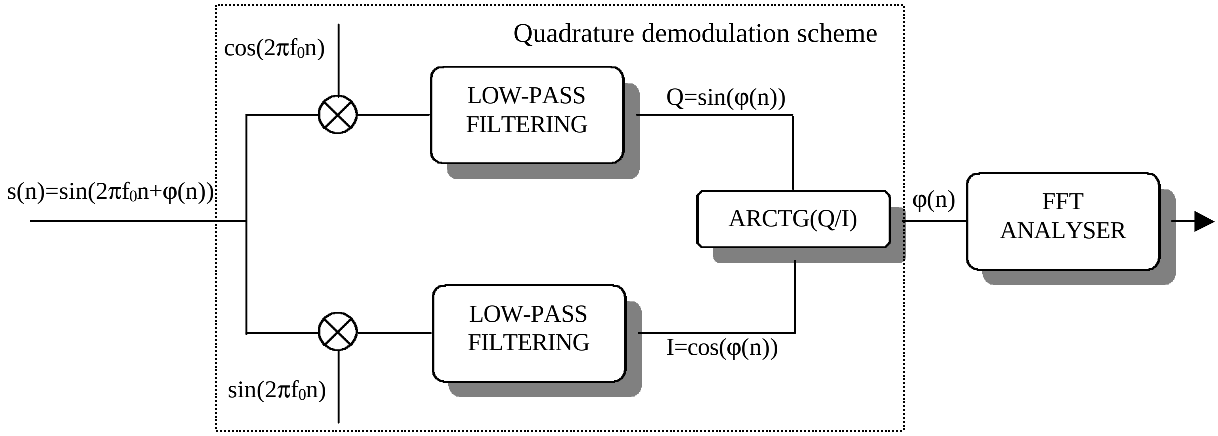


Fig.1 - Block diagram of the old version of the measurement method. The phase fluctuations, *j (n)*, affecting the input signal are evaluated by means of a quadrature demodulation scheme. An FFT analyzer furnishes the power spectrum of *j (n)*.

rate of 1 GS/s is adopted, thus assuring a frequency resolution not better than 1 kHz in any spectral estimate.

The aim of this paper is to overcome the aforementioned problems by means of a suitable technique, which provides for the undersampling of the signal under test and the exploitation of the related aliasing effect.

## 2. THE PROPOSED METHOD

### *2.1 The measurement principle*

As well known, the spectrum of a signal, sampled by an ideal pulse-train running at $f_S$ samples per second, is replicated, in the frequency domain, every $f_S$ hertz. If, in addition, the signal is characterised by a power spectrum concentrated around the carrier frequency $f_0$, both down-conversion and up-conversion can be attained by a proper sampling technique. Specifically, a signal centred at $f_0$ can be converted to infinite frequencies $f_K$ defined by:

$$f_K = \left| \pm f_0 \pm k f_S \right| \quad ; \qquad (1)$$

where $f_S$ is the sample rate and $k$ can assume integer values.

As an example, figure 2 shows the amplitude spectrum of a signal, (130 MHz centre frequency, 20 MHz bandwidth) which is sampled at 100 MS/s (only the positive part of the frequency axis has been reported). It is worth highlighting that replicated spectra are undistorted if two conditions are met: (i) the input signal must be band-limited, and (ii) the sample rate must be greater than twice the bandwidth of the signal. It is clear that no actual signal can strictly match the band-limited requirement. Many signals are, instead, characterised by a significant spectrum *within a compact frequency range* and a negligible one *elsewhere*; these signals can practically be considered band-limited. With regard to an actual sine wave affected by phase-noise, its spectrum consists of a spectral line surrounded by two symmetrical sidebands that have an infinite extent. As a consequence, a partial overlap of replicated spectra always occurs. Fortunately, the phase-noise sidebands quickly roll off as shown in figure 3. Hence the distortion due to overlap can be neglected if there is a sufficient spacing among the replicated spectra; this is particularly true for close-to-the-carrier phase-noise measurement.

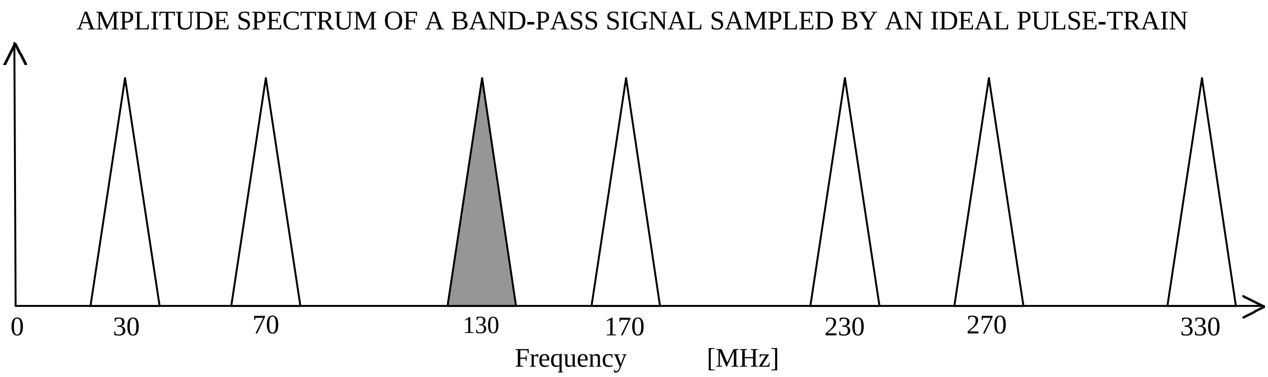


Fig.2 - A schematic frequency domain representation of a signal sampled at 100 MS/s. The bandwidth of the signal is equal to 20 MHz and it is centred at 130 MHz. Only the positive frequencies are taken into account.

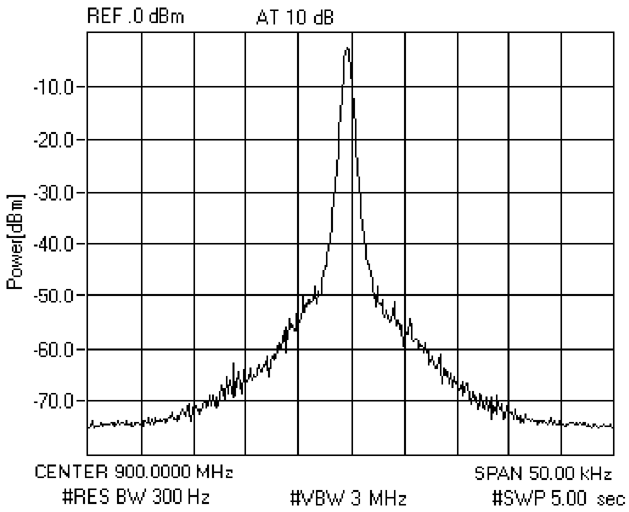


Figure 3. RF spectrum of a sinusoidal carrier (900 MHz nominal frequency) as viewed on a spectrum analyser.

*2.2 Sample rate selection*

The sinusoidal carrier ($f_N$, nominal frequency) provided by the source under analysis is sampled ($f_S$, sample rate) and digitised by means of an ADC. Dealing with under-sampling, an appropriate sample rate has to be selected in order to achieve correct results and best performance. Specifically, if a given frequency resolution, *Äf*, has to be achieved, the sample rate is established according to the following system of equations

$$\begin{cases} \mathrm{mod}\left(\dfrac{f_N}{f_S}\right) = \dfrac{f_S}{4} \\ \dfrac{f_S}{N} \le \Delta f \end{cases} , \tag{2}$$

where '*mod*' stands for remainder after division, and $N$ represents the maximum number of samples that can be collected by the digitiser (memory depth). On the other hand, if a specified baseband frequency interval, *0-B*, has to be analysed, the sample rate is given by

$$\begin{cases} \mathrm{mod}\left(\dfrac{f_N}{f_S}\right) = \dfrac{f_S}{4} \\ f_S \ge 8B \end{cases} . \tag{3}$$

In both cases, thanks to the particular choice of $f_S$, the analysed signal is downconverted to an intermediate frequency equal to $f_S/4$ hertz, and the nearest replica spectrum appears at $3f_S/4$ hertz ($f_S/2$ hertz apart). This is granted by the first equation, which is identical for both systems. With regard to the inequality contained in the system (3), the authors suggest a sample rate at least eight times the bandwidth, $B$, of interest in order to avoid problems due to the partial overlap of replica spectra.

Once the sample rate, $f_S$, has been fixed, the baseband power spectral density of the phase-noise can be obtained up to $f_S/8$ with a frequency resolution of $f_S/N$.

*2.3 The measurement algorithm*

The digitised signal is passed to a quadrature demodulation scheme similar to that shown in figure 1. The signal is then split and mixed to baseband by two orthogonal sinusoidal signals, the frequency of which is equal to the intermediate frequency $f_S/4$. Thanks to the particular ratio between the sample rate and the intermediate frequency (1/4), mixing is just performed by allowing the cosinusoidal and sinusoidal signals to be the repeated data sequences [1,-1,-1,1] and [1,1,-1,-1], which are, respectively, the values of cosinusoidal and sinusoidal functions for phase angles equal to $\pi/4$, $3\pi/4$, $-3\pi/4$, $-\pi/4$ radians when the amplitude is equal to $\sqrt{2}$. Mixing gives rise to both a baseband component and a high frequency component; therefore, a low-pass filtering ($f_S/4$ cut-off frequency) is performed in order to retain only the baseband component. The time-domain evolution of the phase-noise, *j(n)*, is simply retrieved by evaluating the ARCTG of the ratio of the two baseband components. Finally, an FFT algorithm is applied to *j(n)* for evaluating the baseband power spectral density of the phase-noise.

## 3. THE PROPOSED PROTOTYPE

*3.1 Preliminary considerations*

The performance of the proposed method is mostly determined by the characteristics of the adopted ADC, in terms of input bandwidth, effective-bit-number, and sample clock stability. The input bandwidth limits the input frequency range; the effective-bit-number and the stability of the sample clock are responsible of the noise-floor.

With regard to the input bandwidth some considerations have to be made. Specifically, it is desirable to have an input bandwidth wider than half the sample rate. It is so possible to analyse also sinusoidal carriers, the frequency of which does not satisfy the Nyquist condition, without being obliged to use downconverters.

From a theoretical point of view, the inherent noise-floor is mainly determined by two fundamental contributions. The first contribution consists of quantisation noise, the power of which depends on the effective-bit-number of the converter; the power spectral density of this contribution has an approximately constant level versus frequency and it dominates other contributions for offset frequencies far from the analysed carrier. The second contribution refers to the long and medium term stability of the sample clock, which is expressed by the specific phase-noise of the clock itself; it has a typical power-law spectrum and it dominates other contributions for close-to-the-carrier offset frequencies.

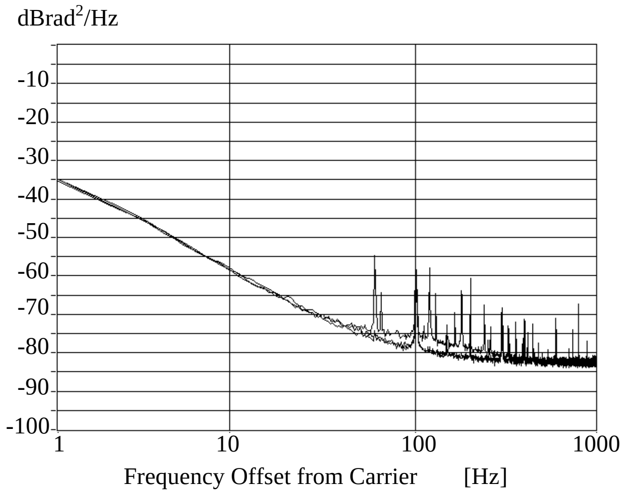


Fig.4 - Phase-noise measurement results furnished by the proposed prototype and related to three sinusoidal carriers at a nominal frequency of 700.025 MHz provided by three different sources. The adopted sample rate has been equal to 100 kS/s and 100.000 samples have been acquired.

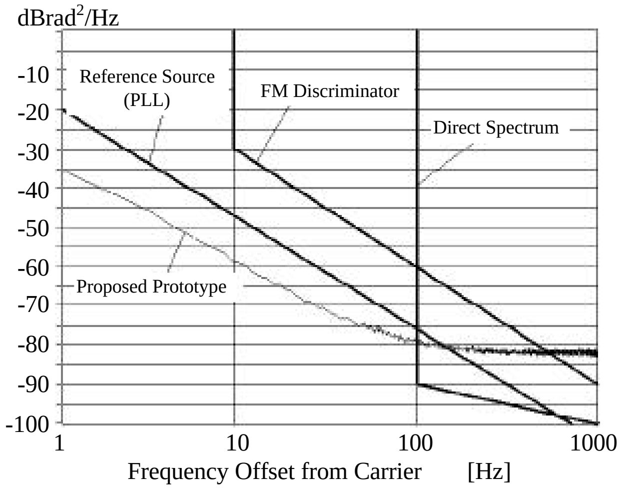


Fig.5 - The noise-floor of the proposed prototype is compared to that characterising other measurement solutions.

*3.2 The hardware*

A prototype system is set up to validate the proposed method by means of experimental tests. It consists of a digital storage oscilloscope (DSO), namely LeCroy LC574A™ (1 GHz bandwidth, 4 GS/s single maximum sample rate, 8-bit nominal vertical resolution), and a personal computer (PC).

They are interconnected by means of an IEEE 488 interface bus. The DSO allows the digitisation of the carrier under test at a sample rate given by either (2) or (3). The acquired samples are passed to the PC, which implements the digital signal-processing algorithm described in the previous section.

*3.3 Experimental tests*

When performing phase-noise measurements, special care has to be taken to ensure that the obtained results reflect the performance of the source under analysis rather than that of the test equipment itself. Hence, the noise-floor of the test equipment has to be determined. In practice, analysing reference sinusoidal signals affected by known phase-noise can do this. If the obtained power spectral density function is greater than the imposed one for each offset frequency, it has to be interpreted as the specific noise-floor of the test equipment. Unfortunately, manufacturers do not always specify close-to-the-carrier phase-noise of typical sources available in laboratory. Therefore, after analysing such sources, it is difficult to understand if the obtained results reflect their performance or reveal the noise-floor of the test equipment.

The same difficulty in establishing the noise-floor of the prototype has been experienced. Specifically, three different sources, HP E4431B ESG-D™ signal generator, HP 8648B™ signal generator, and HP 8643A™ synthetised signal generator, have been adopted for producing reference sinusoidal carriers.

Many tests have been carried out for different frequency values of the sinusoidal carriers, and, for each frequency value, the power spectral densities measured by the prototype and related to the three sources have always resulted very similar to one another. As an example, Fig.4 gives the power spectral densities obtained for a nominal frequency of 700.025 MHz; the adopted sample rate has been equal to 100 kS/s and the number of acquired samples equal to 100,000. It is worth noting that (i) the three traces are superimposed (they are practically indistinguishable), (ii) a very good frequency resolution (1 Hz) has been achieved with values both of the sample rate and memory depth that are very common to be encountered in the data acquisition systems currently available on the market, and (iii) the possibility of a deep investigation of the phase-noise at small offset frequencies is given.

Due to the lack of detailed specifications, especially for close-to-the-carrier phase-noise, of the adopted sources, it is only possible to state that the inherent noise-floor of the proposed prototype cannot be above the traces displayed in Fig.4 (worst-case analysis).

*3.4 Comparison with other measurement solutions*

The previously established worst-case noise-floor of the proposed prototype (all the spurious have been deleted because they are not random noise), has been compared to the typical noise-floor of other measurement solutions, already presented in literature and commonly exploited in the practise. In particular, solutions based on (i) “Direct Spectrum”, (ii) “FM Discriminator” (Delay Line), and (iii) “Reference Source/PLL” measurement techniques have been considered.

From the analysis of Fig.4 it can be noted that the proposed prototype is competitive over the other solutions for close-to-the-carrier phase-noise measurements. The reasons of this success lie essentially on:

- the inability of both “Direct Spectrum” and “FM Discriminator” measurement technique in accurately evaluating the power spectral density of the phase-noise for very low offset frequencies;
- the degradation of the close-to-the-carrier noise-floor of systems based on the “Reference Source/PLL” technique, due to the phase-noise of the auxiliary voltage controlled source they need to provide a reference phase for the phase detector [3]; in particular, the trace given in Fig.4 refers to the HP E5500B™ dedicated phase-noise measurement system operating with LO HP 8642B™ reference source.

## 4. CONCLUSIONS

The paper has presented a digital signal-processing method for phase-noise measurement. It gives the possibility of directly analysing RF sinusoidal carriers by means of the digitisation and processing of the acquired samples, even though the maximum sample rate of the adopted ADC is less than the frequency of the input carrier. Moreover, it assures a good frequency resolution without requiring a large memory depth of the data acquisition system used.

Experimental tests have assessed the reliability of the method and highlighted the promising performance of a prototype system.

It is worth noting that some improvement of the performance exhibited by the presented prototype can be attained by means of a dedicated hardware. In particular, to reduce the contribution of the quantisation noise to the overall noise-floor of the prototype, high resolution ADC’s could be used. This is made possible by the low sample rates required by the method. Further benefits could also be attained if a very stable clock is used to provide the effective sample rate; a very low phase-noise sample clock, in fact, does not affect the close-to-the-carrier noise-floor excessively. Moreover, the sample clock required has to operate at a frequency much less than that of the carrier under test. This also represents a significant advantage over dedicated measurement systems, which need a reference source operating at the same frequency of the analysed carrier.

## REFERENCES


[1] K.Feher, “Telecommunications, Measurements, Analysis and Instrumentation”, Prentice-Hall, Inc. Englewood Cliffs, New Jersey 1987.

[2] S.Bregni, “Clock Stability Characterization and Measurement in Telecommunications”, *IEEE Trans. on Instr. and Meas.*, December 1997, vol.46, No.6, pp.1284-1294.

[3] “Phase Noise Measurement Seminar”, *Hewlett-Packard* literature number 5968-3734EE.

[4] D.Cosart, L.Peregrino, A.Tambe, “Time Domain Analysis and Its Practical Application of Phase Noise and Jitter”, *IEEE Trans. on Instr. and Meas.*, August 1997, vol.46, No.4, pp.1016-1019.

[5] L.Angrisani, M.D’Apuzzo M.D’Arco “A Digital Signal Processing Approach for Phase Noise Measurement”, *IEEE Trans. on Instr. and Meas.*, August 2001 *in press*.

[6] E.Rubiola, V.Giordano, J.Groslambert, “Very high frequency and microwave interferometric phase and amplitude noise measurement”, *Review of Scientific Instruments*, vol.70, No.1, Jan. 1999, pp.220-225.

[7] N.Majurec, R.Nagy, G.Sisul, “PC based Microwave Phase-Noise Measurement System”, Proc. of *IEEE IMTC/2001*, Budapest, Hungary, May 21-23, 2001, vol.2, pp.540-543.