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TRACEABILITY OF BRIDGE AMPLIFIERS IN TARED MEASUREMENT APPLICATIONS

A. Brüge

Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany, andreas.bruege@ptb.de

Abstract:
The influence of the primary voltage ratio standard uncertainty on strain gauge measurements, especially at secondary levels like torque wrench calibration, is discussed. A method is presented to overcome its 1/x behaviour, which affects the uncertainty budget at lower ratios for the usual point-by-point transfer of uncertainty.

Monte Carlo simulations of fitted correction values reduce the relative expanded contribution of the traceability uncertainty to less than $1 \cdot 10^{-5}$ for tared measurements with bridge amplifiers in comparison to five times more for point-by-point traceability.

Considering typical further uncertainty contributions, a tared measurement of voltage ratios is possible with a combined relative expanded uncertainty of less than $1.8 \cdot 10^{-5}$.

Keywords: voltage ratio, traceability, bridge amplifier, uncertainty, linearity.

1. INTRODUCTION

Due to mechanical challenges, torque wrench calibrations are often connected to higher uncertainties than those for torque transducers. Therefore, for many laboratories concerned with such calibrations, it is reasonable to use reference calibration devices instead of deadweight machines. Additional uncertainties then arise because of the traceability of the bridge amplifier which is employed to control the generated torque in these devices.

Furthermore, for reference calibration devices, it is usual to operate several reference transducers with one amplifier in order to cover a wide measuring range. A traceability calibration of one of these reference transducers could then not include the amplifier without putting the device out of service in the meantime. Thus, a further contribution of voltage ratio traceability according to the amplifier interchangeability has to be taken into account.

The traceability of bridge standards and bridge amplifiers to the primary standard voltage divider is associated with a constant absolute uncertainty U_d over the range of operation [1], [2]. This leads to high relative uncertainties for smaller voltage ratios V if U_d is applied point-by-point at the calibration steps.

2. TRACEABILITY CONCEPT

Bridge amplifiers are traceable to the primary standard of voltage ratio via bridge standards which are transfer standards consisting of a switchable divider network. In the calibration of bridge standards, the readings of the primary standard and of the bridge standard N' are compared, and a correction value K_{BN} for each calibrated voltage ratio V is determined which delivers the corrected reading N of the bridge standard:

$$N(V) = N'(V) + K_{BN}(V) \quad . \tag{1}$$

The calibrated bridge standard can then be used to simulate a detuned Wheatstone bridge at the input of a bridge amplifier. Similarly, a comparison between the corrected reading of the bridge standard N and the reading of the amplifier A' delivers correction values K_V of the amplifier for the calibrated voltage ratios:

$$K_V(V) = N(V) - A'_V(V) \quad , \tag{2}$$

which leads to the corrected reading of the bridge amplifier:

$$A_V(V) = A'_V(V) + K_V(V) \quad . \tag{3}$$

Finally, the calibrated bridge amplifier can be used in a calibration by measuring voltage ratios of real loaded strain gauges. Here, the correction values K_V can be used to correct the readings of the amplifier or, if the values of K_V do not exceed the required uncertainty budget, they can be used as uncertainty contributions.

3. DETERMINATION OF K_{BN}

Since a bridge standard can constitute only stepped values of voltage ratio, a calibration arbitrarily covering the bridge amplifier range is not directly possible. But applying the voltage ratio steps of the bridge standard point-by-point is inadvisable, because their constant absolute amount of traceability uncertainty $u(K_{BN})$ would lead to high values of relative uncertainty at lower voltage ratios. Furthermore, reference calibration devices need traceability in a continuous manner, so a regression of the stepped values of the national standard is indispensable.

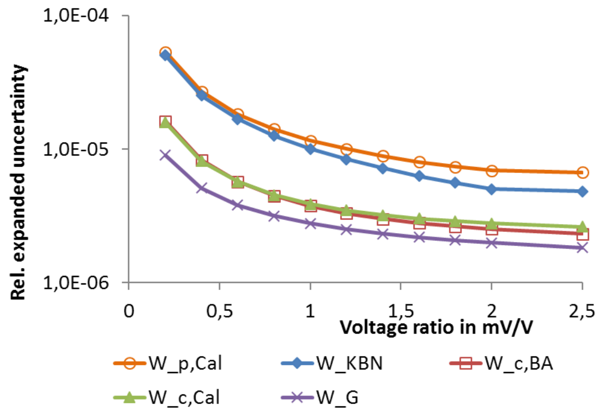

Figure 1: Relative expanded measurement uncertainties (k=2) under different conditions. The uncertainty of a tared measurement with point-by-point traceability of the voltage ratio $W_{p,Cal}$, the uncertainty of the point-by-point voltage ratio traceability W_{KBN}, the combined uncertainty of absolute bridge amplifier measurements $W_{c,BA}$, the combined uncertainty of a tared voltage ratio measurement $W_{c,cal}$ and the uncertainty of a brigde standard traceability by MCM, as described in the text, are shown.

A method is therefore proposed to determine the values of $K_{BN}(V)$ and their uncertainties by deploying the entity of the voltage ratios V_i calibrated by the primary standard. It consists of a linear fit through the measured data by a simulation of the uncertainties of the bridge standard's calibration data (Monte Carlo method, MCM), which assigns each $K_{BN}(V_i)$ to a normally distributed value of uncertainty $\delta K_{BN}(V_i)$ with a standard deviation in the amount of $u(K_{BN})$. Subject to the condition that the bridge standard is a linear instrument, there is a best-fit line G for each simulated set of data points $K_{BN}(V_i) + \delta K_{BN}(V_i)$:

$$G(V) = K_{BN}(V) = g_0 + g_1 \cdot V \qquad . \qquad (4)$$

In the calculated sample of best-fit lines, which turned out to be normally distributed, an regression line G_{expt} can be found with expectation values of the slope $E(g_1)$ and of the intercept $E(g_0)$ and their standard deviations $\sigma(g_1)$ and $\sigma(g_0)$. Thus, the absolute uncertainty value $u(K_{BN})$ is replaced by $\sigma(g_1)$ and $\sigma(g_0)$. Since $\sigma(g_1)$ is a relative quantity and constant for the entire range of voltage ratios and $\sigma(g_0)$ is less than a third of the amount of $u(K_{BN})$, the contribution of the primary voltage ratio standard uncertainty W_{KBN} has been reduced considerably to W_G (Figure 1). As another benefit, a conclusion about values between the data points is possible.

The determination of the best-fit line can also be carried out analytically according to the GUM [5]. The results of simulations and of analyses are close for the line parameters but not for their uncertainties. The effort for simulations is much higher than for GUM analyses, but the latter are only valid for the case of negligible uncertainties dedicated to the data points. This condition is obviously not fulfilled in this case, so the uncertainties of the GUM method only reflect the influence of the regression error. An estimation of the

Table 1: Parameters of the determination of K_{BN} or K_V determined with different methods as described in the text.

Method	$E(g_0)$ in 10^{-6} mV/V	$\sigma(g_0)$ in 10^{-6} mV/V	$E(g_1)$ in 10^{-6}	$\sigma(g_1)$ in 10^{-6}
MCM	-25.430	1.562	-9.108	1.208
GUM	-25.431	0.320	-9.099	0.238
GUM*	-25.431	1.478	-9.099	1.462
	$E(h_0)$ in 10^{-6} mV/V	$\sigma(h_0)$ in 10^{-6} mV/V	$E(h_1)$ in 10^{-6}	$\sigma(h_1)$ in 10^{-6}
MCM_G	-21.299	0.578	-14.045	0.612
GUM_G	-21.303	0.841	-14.038	0.710
GUM_G*	-21.303	1.007	-14.038	0.930
MCM_c	-21.298	0.766	-14.500	0.969
$MCM_{c.tar}$	0	0	-13.026	0.925

missing influence of the line parameter uncertainties can be found regarding the GUM analyses as an averaging with normal distribution ($u(K_{BN})/\sqrt{n}$). The combinations of both contributions are given in Table 1 where they are referred to as GUM*.

4. DETERMINATION OF K_V

Similar to the determination of K_{BN} the corrections K_V for the bridge amplifier are found by the MCM using

$$H(V) = K_V = h_0 + h_1 \cdot V \qquad . \qquad (5)$$

Here, the underlying uncertainties $u(K_V)$ of the data points are given by the contributions of K_{BN} discussed in the section above. For a complete uncertainty budget, additional contributions due to the properties of the bridge amplifier are to be taken into account. This can be done by considering these contributions in combined values of $u(K_V)$, which is discussed in Section 8.

The results of the MCM calculation considering $\sigma(g_1)$ and $\sigma(g_0)$ are given in Table 1 where they are referred to as MCM_G. The corresponding estimations according to the GUM are given as GUM_G and GUM_G*. Apparently, the uncertainty of the GUM_G regression is dominated by the residual error here; thus the estimation already reaches the level of MCM_G without regarding the uncertainty of the data points, which is done in GUM_G^*.

When the amount of K_V is small enough not to compromise the uncertainty budget of a measurement, the application of K_V is often seen as dispensable and the deviation of the amplifier shall be considered as a systematic uncertainty contribution. The uncertainty contribution caused by this procedure is given in Figure 2 as $W(K_V)$ together with the systematic error due to K_{BN} referred to as $W(K_{BN})$ and, for comparison, the result of a complete uncertainty budget for tared measurements $W_{c,cal}$. Obviously, it is recommendable to apply the correction K_{BN} in any case, whereas K_V would not carry so much weight in a per mille uncertainty budget.

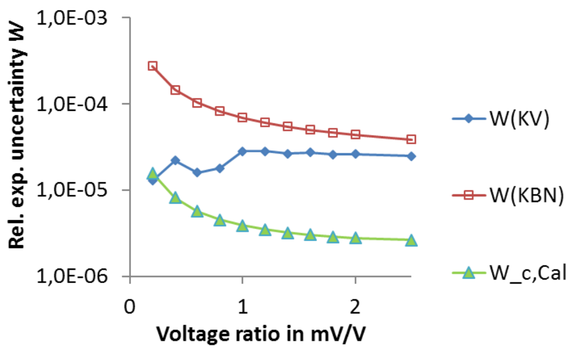

Figure 2: Relative expanded uncertainties (k = 2) due to the corrections K_{BN} (red) and K_V (blue) if they were not applied for compensation but taken into account as systematic contributions to the measurement uncertainty. For comparison, the combined relative expanded uncertainty out of Figure 1 is shown (green).

For more ambitious measurements like key comparisons, however, it is advisable to apply the amplifier correction and to take into account the uncertainty of this correction.

5. APPLICATION OF BRIDGE AMPLIFIERS

The voltage ratio response $V_W(M)$ of a torque wrench to a torque load M can be represented by a polynomial of the third degree:

$$V_W(M) = d_0 + d_1 M + d_2 M^2 + d_3 M^3 \quad . \quad (6)$$

The measuring chain consisting of the torque wrench and the bridge amplifier transforms the voltage ratio $V_W(M)$ into a reading $A(M)$. According to (3) and (5), for a linear amplifier, the corrected reading can be written as

$$A(M) = V_W(M) + h_0 + h_1 M \quad . \quad (7)$$

In torque calibrations, signals are always related to a zero value. Usually a sequence with measurements $A(M_i)$ at torque steps M_i is performed which begins and ends with a practically load-free measurement $A(M_0)$. A shift by the value of $A(M_0)$ delivers the tared readings of the amplifier A_{tar}:

$$A_{tar}(M_i) = A(M_i) - A(M_0) \quad . \quad (8)$$

With (6) and (7), the tared reading becomes

$$A_{tar}(M_i) = d_1 \cdot (M_i - M_0) + d_2 \cdot ({M_i}^2 - {M_0}^2) + d_3 \cdot ({M_i}^3 - {M_0}^3) + h_1 \cdot (M_i - M_0) \quad . \quad (9)$$

This formula is free of zero-order terms and in consequence a tared measurement is not influenced by intercepts of the best-fit line and of the polynomial involved in the measurement of its two parts. Therefore, the expanded combined uncertainty contribution of the bridge amplifier traceability in tared measurements is restricted to $\sigma(h_1)$ and is a constant relative value of about $0.6 \cdot 10^{-6}$ in the measurement range (Table 1, MCM_G).

Hence, the uncertainty budget of a bridge amplifier, if used as an absolute instrument, contains both the contributions of h_0 and h_1. These coefficients and their uncertainties are achieved by a best-fit line with a loose intercept. After all, the standard deviations of the fitting line residuals also have to be considered.

Figure 3: Three-state bridge as a voltage ratio transfer standard.

If used as a differential instrument, its uncertainty budget should rather include both the contributions of h_1 and the residuals twice. But when calculated with tared data points and with a fixed intercept (Table 1, $MCM_{c,tar}$), these contributions of the zero measurement were reduced to zero. Only one contribution of h_1 and the residuals each is then necessary (Table 3, column $u(K_{V,tar})$).

In practice, the representation of the sensitivity by a polynomial is always based on the tared values $\Delta M_i = (M_i - M_0)$ of the calibration. Therefore, (**6**) becomes

$$V_W(\Delta M_i) = d_0 + d_1 \Delta M_i + d_2 {\Delta M_i}^2 + d_3 {\Delta M_i}^3 \quad (10)$$

and (9) then becomes

$$A_{tar}(M_i) = d_1 \cdot \Delta M_i + d_2 \cdot {\Delta M_{i_i}}^2 + d_3 \cdot \Delta M_i + h_1 \cdot \Delta M_i \quad . \quad (11)$$

In this way, the complete traceability chain is based on tared values and the deviation because of $({M_i}^n - {M_0}^n) \neq (M_i - M_0)^n$, n>1 vanishes.

6. LINKING BRIDGE AMPLIFIERS

If each laboratory uses its own amplifier in comparisons, the described traceability of differential measurements is not adequate to evaluate the equivalence of the amplifiers. With the help of a transfer three-state bridge (Figure 3), steps of 0 mV/V to 2 mV/V and 0 mV/V to -2 mV/V could be measured at each amplifier. In this manner, the isolated differential calibrations of the participants can be brought together on a joint basis.

To compensate for the influence of laboratory conditions, the relative temperature coefficient of the transfer bridge was achieved to $-1.9 \cdot 10^{-5}$ K^{-1} and the amount of the relative humidity coefficient, to smaller than $1 \cdot 10^{-7}$ $\%_{rH}^{-1}$. The relative uncertainty of the temperature coefficient was found to be less than $1 \cdot 10^{-6}$ K^{-1}. Although the steps are performed with intervals of only a few seconds, taring cannot reduce the impact of temperature dependency in this case, because the temperature sensitivity of the voltage ratio zero value was found to be much smaller than for the ± 2 mV/V steps.

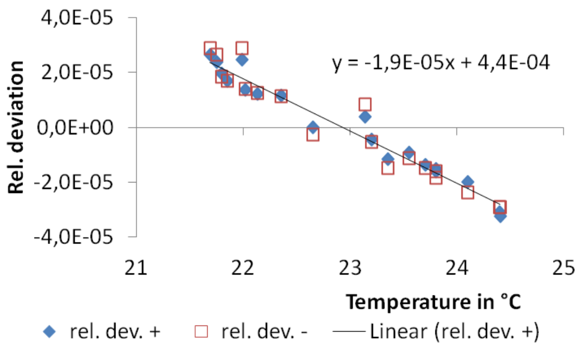

Figure 4: Relative deviations of the transfer bridge voltage ratio due to temperature variation.

In the first tests at bridge amplifiers (HBM DMP40), such a transfer bridge delivered calibrations with a relative uncertainty of about $3.2 \cdot 10^{-6}$, including a relative uncertainty of the correction of ambient influences of less than $2 \cdot 10^{-6}$ at laboratory conditions (Figure 4).

With these properties, a transfer bridge compensating for ambient influences would be adequate for key comparisons at the level of transfer torque wrenches.

Table 2: Considered uncertainty contributions in the traceability steps towards tared application of bridge amplifiers (BA) via a bridge standard (BN). Starting with the determination of K_{BN} {a}, combined $\delta u(K_V)$ {b} are calculated for use in the MCM, which results in the traceability for the bridge amplifier corrections $u(K_V)${c}. The intercept-fixed MCM delivers traceability for tared bridge amplifier applications $u(K_{V,tar})${d}.

Source	u_i (k=1) in 10^{-6} mV/V	Used in {x}
K_{BN}, primary standard	5.00/6.00	a
g_0	1.56	b
g_1	$1.21 \cdot V_i$	b
Residual standard dev. G	0.46	a, b
Long-term drift BN	$1.25 \cdot V_i$	b
Humidity correction, BN	$0.14 \cdot V_i$	b
Temp. correction, BN	$0.10 \cdot V_i$	b
Temp. correction, BA	$1.18 \cdot V_i$	b
Humidity correction, BA	$0.78 \cdot V_i$	b
Reproducibility, BA	$0.70 \cdot V_i$	b
Resolution BA	0.29	b
Repeatability, BA	$0.21 \cdot V_i$	b
Short-term zero drift, BA	0.29	b
h_0	0.77	c
h_1	$0.97 \cdot V_i$	c
$h_{1,tar}$	$0.93 \cdot V_i$	d
Residual standard dev. G	1.41	c, d
Long-term drift, BA	$0.62 \cdot V_i$	d

7. TESTING LINEARITY

Table 3: Relative temperature coefficient C_T and relative humidity coefficient C_{rF} of bridge instruments [4].

Instrument	C_T in 10^{-6} /K	C_{rF} in 10^{-6} /$\%_{rF}$
bridge standard BN100	0.34	0.12
bridge amplifier DMP40	3.0 ...9.0	0.14 ...0.90

An important assumption for the presented method is the linearity of the bridge instruments. Using the bridge standard in the traceability procedure delivers values of the linearity at increments of 0.1 mV/V. The relative standard deviations of the best-fit line residuals for bridge amplifiers are typically smaller than $2 \cdot 10^{-6}$, which is, nevertheless, a dominant contribution to the uncertainty budget.

Additional investigations could be carried out with a combinatorial method using a compensated resistor network (CRN) [5]. Initial results with this method seem to allow the evaluation of the linearity of a bridge instrument only in a higher range of some 10^{-5}, but the density of the tested voltage ratios is, depending on the number of resistors in the network, much higher than with a bridge standard. Moreover, this linearity test only needs one traceable calibration value.

A simple test with a self-manufactured CRN under different environmental conditions yielded no significant influence of ambient air humidity in the range of 30 $\%_{rH}$ to 80 $\%_{rH}$. The relative influence of temperature within the range of 21.5 °C to 24.5 °C was found to be about $2 \cdot 10^{-5}$/K , which is significantly higher than for usual bridge standards and bridge amplifiers. The CRN measurements were connected with a standard deviation of about $2.5 \cdot 10^{-6}$ mV/V, which may lead to a relative influence of $2.5 \cdot 10^{-5}$ for the smallest voltage ratio of the CRN.

Because we used resistors which are customary in trade in the CRN, improvements of these parameters should be possible through the application of high precision resistors with low temperature coefficients.

8. COMBINED UNCERTAINTY BUDGET

As mentioned above, additional uncertainty contributions are to be taken into account in the procedure leading towards the traceable application of bridge amplifiers. Ambient conditions in particular can influence the response of bridge instruments significantly [4]. The coefficients of the examined amplifiers were shown to cover a wide range of values (Table 2). Therefore, for high precision applications of bridge amplifiers, the correction for ambient conditions seems to be essential.

The uncertainty sources and their standard uncertainty amounts taken into account in the traceability calibration of the bridge amplifier and in its application are listed in Table 3. For this paper, the contributions were employed as combined values of $u(K_V)$ in the MCM analysis to achieve the regression line G. Then G and the residual errors of the regression line are combined to the uncertainty $W_{c,BA}$ of the bridge amplifier for absolute measurements.

For differential measurements, an intercept-fixed MCM regression, as described in Section 5, delivers the combined uncertainty $W_{c,cal}$. Both uncertainties are given in Figure 1. Ultimately, the relative contribution of the amplifier to the uncertainty of a tared measurement is smaller than $1.6 \cdot 10^{-5}$ within a reasonable range of voltage ratios.

In comparison to these results of the MCM, the values for a tared measurement under the conditions of point-by-point traceability of the voltage ratio are shown and are referred to as $W_{p,Cal}$, reaching up to $5.3 \cdot 10^{-5}$.

9. CONCLUSION

The use of bridge amplifiers in calibration applications is connected with uncertainties due to traceability to the primary standard. With best-fit lines and Monte Carlo simulations it is possible to calculate the impact of these uncertainties combined with additional application uncertainties to a relative expanded contribution of less than $1.6 \cdot 10^{-5}$ which is about three times less than with point-by-point applications.

Although they consist of two measurements, the measurement uncertainty results in about the same value for differential measurements with bridge amplifiers. This is possible with an intercept-fixed MCM regression, which enables a reduction of the influence of the zero measurement. The presented method is thus suitable for measurements like torque wrench calibration key comparisons.

The linearity of the instruments is a crucial requirement. Methods available for linearity validation work in the same uncertainty range as the presented uncertainty budgets, but can reach only discrete values of voltage ratio. Nevertheless, it has become apparent that the linearity residuals dominate the uncertainty budgets of differential voltage ratio measurements and should be considered carefully.

Combinatorial methods could reach arbitrary ratios in principle, but have not been investigated in relation to their measurement uncertainty to date.

10. REFERENCES

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