Investigating mathematical models of waviness measurements of cylindrical elements by the V-block method through computer simulations

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Abstract:
The paper relates to the problem of an adaptation of V-block methods to waviness measurements of cylindrical surfaces. It presents fundamentals of V-block methods and a principle of their application. V-block methods also called reference methods can be successfully used to measure roundness and waviness deviations of large cylinders utilized in paper industry, shipping industry or in metallurgy. The paper describes a methodology of development of a mathematical model of roundness and waviness measurements by the V-block method. Next, a computer simulation was carried out that aimed at checking if proposed model is correct. Another aim of the simulation was to verify the concept of the adaptation of the V-block method to waviness measurements of cylindrical surfaces. . The results of the simulations show that if parameters of the V-block method are correctly chosen then a projection accuracy of both roundness and waviness is high.

Keywords: V-block methods, roundness, waviness measurement, detectability coefficient

1. INTRODUCTION

Nowadays, modern industries faces great demands to control process quality, meet product specifications, correspond to customer needs and assess their R&D activities. As a result, the development a proper measurement methodologies for product evaluation becomes a critical issue [1]. Particularly important is the development of measuring methods that can be used to perform in-situ measurements directly on a machine tool, often in unfavourable industrial conditions.

Cylindrical element are very important group of mechanical parts, because about 70% of all engineering an axis of rotational symmetry [2]. The surface or texture characterization is an important component of cylindrical surface description. From a structural perspective, any surfaces can be as the overlap three types of irregularities, form and position error (roundness error for cylindrical surfaces) waviness and roughness [1].

Roundness and waviness error of rotating parts generates vibrations. This phenomenon is very dangerous, because it reduces service cycle of mechanical components and causes serious malfunction. For this reason, roundness deviation and waviness of cylindrical surfaces should be measured.

Traditionally roundness and waviness of cylindrical part are characterized using two main methods: a radius change method also called non-reference method and a V-block method known as reference method. The radius change method is based on two types of measuring instrument: ones employing a rotary sensor (Fig. 1a), and ones employing a rotary table (Fig. 1b). This measuring method is widely accepted in laboratories and industrial facilities, because it is simple and accurate. It is know that these methods, despite being very simple, are ineffective in measuring the large-size cylinders.

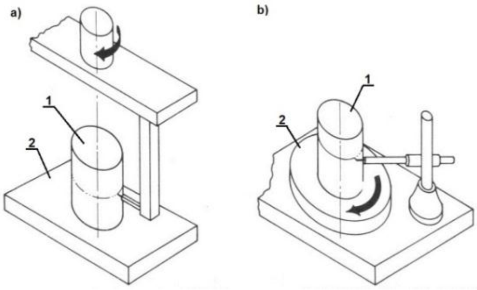

Fig. 1: Radial roundness measurement: (a) using the rotary sensor, (b) using rotary table [3]

As shown in Fig. 1 in non-reference method workpiece (1) should be placed on the measuring table (2) of the instrument. In many branches of industry such as paper industry [4], metallurgical industry, shipbuilding, there are large-size cylinders that cannot be placed on measuring table.

The V-block method meets this requirements. This methods can be successfully used to measure roundness and waviness deviations of large cylinders and to perform in-situ measurements directly on a machine tool.

2. MATHEMATIC MODEL OF V-BLOCK WAVINESS MEASUREMENTS

2.1 Characteristics of reference methods

Reference methods are two-points, three-points and n-points. In the V-block method there are base points (S_2, S_3) and point of measurement - S_1.

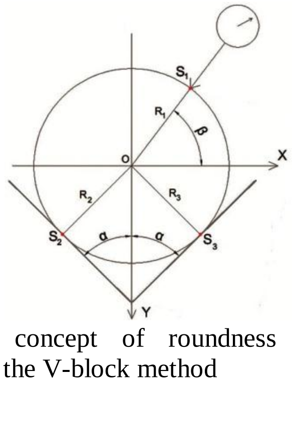

Fig. 2: The concept of roundness and waviness measurements by the V-block method

Their position with regard the assumed coordinate system is determined by two angles: α and β.

The angle α is the angle between the tangents up to a supports of the measured object, whereas the angle β is the angle between the direction of measurement and the co-ordinate axis X (Fig. 2). The values of α and β are the angular parameters, which are responsible for detecting particular harmonic components of measured profile.

In practice values of signal read by the sensor is depend on the deviations in the point of measurement S_1 and base points S_2, S_3. As a result of this profile *F(φ)* measured by V-block method does not coincides with real profile *R (φ)* (see Fig. 3).

This was basic problem of V-block method application. It was necessary first determine the number of the predominant harmonic components, and then to calculate the approximate value of the roundness deviation ΔZ.

$$\Delta Z \cong \frac{\Delta F}{K_n}, \qquad (1)$$

where ΔF – measured deviation, K_n – coefficient of detectability for n-th harmonic.

Coefficient of detectability is the function of the method parameters – angles α and β, as well as the number of n determining for regular profiles, the so-called n-lobbing.

Because V-block methods had low accuracy and therefore they used to be applied to the approximate roundness measurement, a mathematical model was development to increase measurement accuracy of V-block method. Further information can be found in Ref. [5], which deals with applicability of the mathematical models to V-block waviness measurements.

3. COMPUTER SIMULATION

The simulation was carried out with the use of Matlab software to obtain procedures and functions for checking if proposed model and angular parameters are correct. Another aim of the simulation was to verify the concept of the adaptation of the V-block method to waviness measurements of cylindrical surfaces.

The computer simulation presented in this article involves the following three steps: (I) generate profile *R(φ)* which correspond to the real roundness and waviness profile of the workpiece, (II) generate profile *F(φ)* obtained directly from measuring sensor, (III) calculated profile $R_p(\varphi)$ from profile *F(φ)* thought its mathematical transformation.

First of all transformation was conducted for 2-50 UPR (undulation per revolution), and next for 16-50 UPR.

3.1 Generation of real profile R(φ) and profile measured by sensor F(φ)

A real profile *R(φ)* generated using computer simulation consist 4,6,11 harmonic components, which covers roundness deviation and 18, 25, 32, 40, 43, 47, 50 harmonic components, which represent waviness deviation of cylindrical surfaces. This profile is described by Eq. (2).

$$R(\varphi) = R_0 + 0.001 * (\sin(\varphi) + \cos(\varphi) + 3 * \cos(4\varphi) + 2 * \sin(8\varphi) + \cos(11\varphi) + \sin(18\varphi)\, 2\cos(25\varphi) + 2\sin(32\varphi) + \cos(40\varphi) + 2\sin(43\varphi) + 1.5\sin(47\varphi) + \cos(50\varphi)). \qquad (2)$$

where: R_0=25mm - the nominal cylinder radius,
$\varphi = \frac{2\cdot\pi\cdot n}{N}$ - the sampling angle, where n=0,1,2,…,N.

When we have real profile *R(φ)*, in the next step we calculate the profile *F(φ)*, measured by the sensor.

Authors of works [6, 7] show that the V-block methods can be applied to measurements of cylindricity, if one performs a mathematical transformation of the measured profile into a real one. Therefore, mathematical relationships presented in [6, 7] were used to obtain profile *F(φ)*.

$$F(\varphi) = R(\varphi+\beta) + E_x(\varphi)cos\beta + E_y(\varphi)sin\beta$$
$$= R(\varphi+\beta) - \frac{1}{2}R(\varphi+\alpha)\left[\frac{cos\beta}{cos\alpha} + \frac{sin\beta}{sin\alpha}\right]$$
$$-\frac{1}{2}R(\varphi+\pi-\alpha)\left[-\frac{cos\beta}{cos\alpha} + \frac{sin\beta}{sin\alpha}\right]. \qquad (3)$$

where: α=60°, β=55°.

Profile *F(φ)* was calculated from the Equation (3) for given method parameters, i.e α=60°, β=55°.

Research presented in [8] proved that such combination of angles allows users to obtain high measurement accuracy of V-block waviness measurement, because in this case all detectability coefficients in the range K_{16}-K_{50} are not equal to zero and they reach satisfactory values.

In order to show two profiles, namely real profile *R(φ)* and profile *F(φ)* measured by sensor, comparison of their profiles in Cartesian co-ordinates was carried out in Fig. 3.

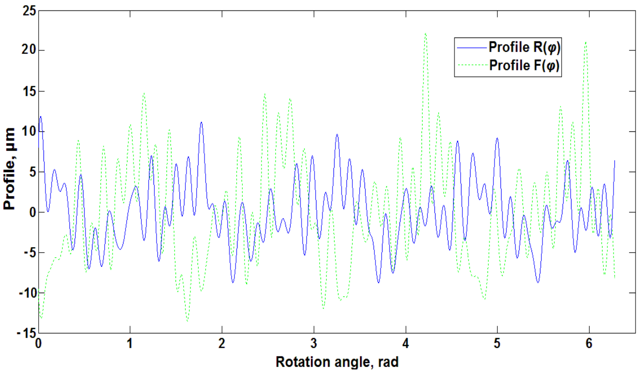

Fig. 3: Comparison of the profile *F(φ)* measured by sensor with real profile *R(φ)*

As we can see in Fig. 3, that there is difference between the inspected profiles. Therefore deviation ΔF measured by sensor does dot coincides with real roundness and waviness deviation ΔZ. For that reason in order to increase a measurement accuracy of V-block methods it is necessary to transform the measured profile *F(φ)* into the reconstructed profile $R_p(\varphi)$ using following relationships.

3.2 Transformation measured profile F(φ) into reconstructed profile $R_p(\varphi)$

Let us see Eq 3. In this equation only the profile *R(φ)* is unknown. Thus, the equation should be solved in relation to *R(φ)*. The problem, however, can be simplified when the measured and real profiles are presented in the form of a complex Fourier series.

Let $\widehat{F_n}$ and $\widehat{R_n}$ be n-th components of expansion of the profiles $F(\varphi)$ and $R(\varphi)$ in a complex Fourier series n= - ∞,…,-1,0,1…,∞, that is

$$R(\varphi) = \sum_{n=-\infty}^{\infty} \hat{R}_n e^{in\varphi} \; , F(\varphi) = \sum_{n=-\infty}^{\infty} \hat{F}_n e^{in\varphi} \qquad (4)$$

Then, we get from Eq. (3):

$$\widehat{F_n} = \widehat{R_n} \cdot \widehat{K_n} \qquad (5)$$

where $\hat{K}_n$ is so-called coefficient of detectability for the n-th harmonic of profile defined by

$$\hat{K}_n = e^{in\beta} - \frac{1}{2} e^{in\alpha} \left[\frac{cos\beta}{cos\alpha} + \frac{sin\beta}{sin\alpha}\right] - \frac{1}{2}(-1)^n e^{-in\alpha} \left[-\frac{cos\beta}{cos\alpha} + \frac{sin\beta}{sin\alpha}\right] \stackrel{\text{def}}{=} \frac{\hat{R}_n}{\hat{F}_n} \qquad (6)$$

Application of the above relationships in computer procedures allow performed transformation the measured profile $F(\varphi)$ into reconstructed profile $R_p(\varphi)$.

Firstly, simulation was performed in range 2-50 UPR, which covers roundness and waviness components. Figure 4 shows superimposed real profiles R(φ) and reconstructed $R_p(\varphi)$ in Cartesian co-ordinate. The diagram in Fig. 5 represent the bar chart of amplitudes of subsequent harmonic components (for range 1-50) of the inspected profiles.

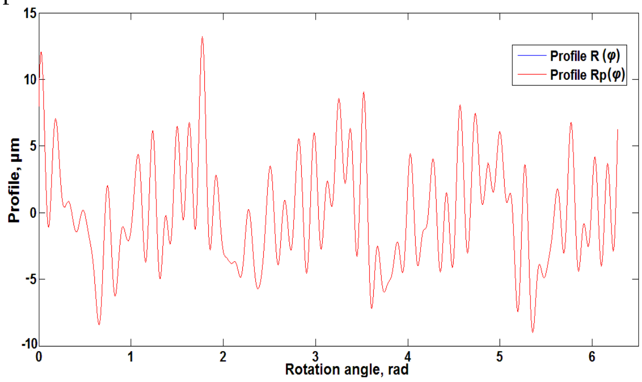

Fig. 4: Superimposed reconstructed profile $R_p(\varphi)$ with real profile $R(\varphi)$

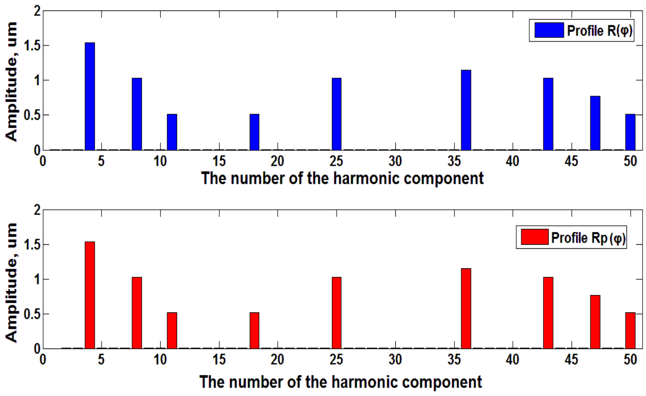

Fig. 5: The bar chart of harmonic components for the profiles: upper blue chart – real profile $R(\varphi)$, lower red chart - reconstructed profile $R_p(\varphi)$

As we can see in Fig. 4 and Fig. 5 reconstructed profile $R_p(\varphi)$ practically overlaps with real profile $R(\varphi)$. The compatibility of compared profile is very high. In order to represent difference between two profiles, the diagram in Fig. 6 represent the difference between profiles illustrated in Fig. 4

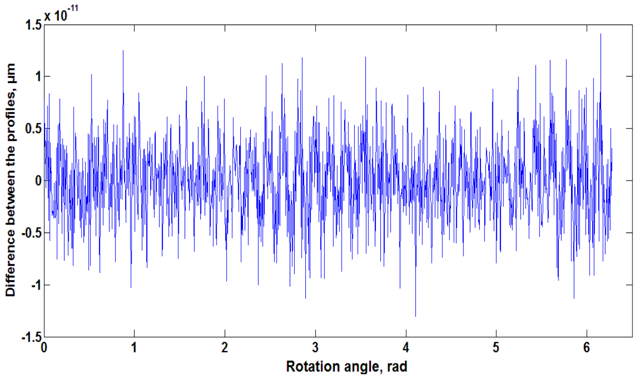

Fig. 6: The difference between the profiles $R(\varphi)$ and $R_p(\varphi)$

The maximum value of the difference between considered profiles equal $1{,}41 \times 10^{-11}$µm (see Fig. 6). Therefore very high level of compliance between profiles was achieved. The results of simulation show that proposed parameters of V-block method and mathematical model can be successfully used to measured roundness and waviness profile in range 2-50 UPR.

The next step in computer simulation comprised the transformation the measured profile $F(\varphi)$ into the reconstructed profile $R_p(\varphi)$ in range 16-50 UPR which covers only waviness components [9].

In order to application V-method to waviness measurement, the real profile $R(\varphi)$ and profile $F(\varphi)$ measured by sensor were filtered to delete roundness components and leave only waviness components in range 16-50 UPR (see Fig. 8).

The profiles were filtrated using harmonic analysis with Fast Fourier Transformation.

Then profile $F(\varphi)$ was transformed into reconstructed profile in range 16-50 UPR, which covers waviness components. Figure 7 present comprised two profiles in Cartesian co-ordinate, while Figure 8 shows the bar chart of amplitudes of harmonic components for range 16-50.

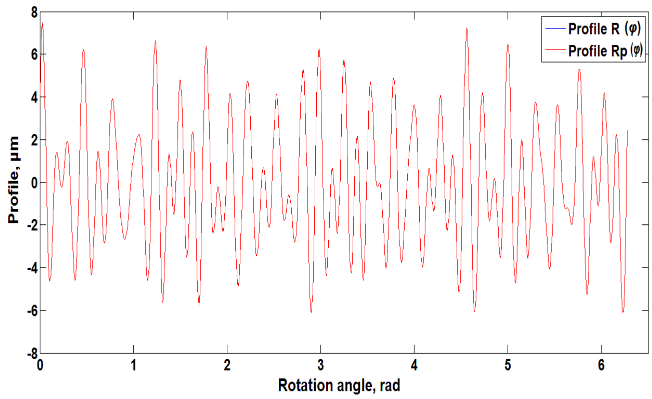

Fig. 7: Superimposed reconstructed profile $R_p(\varphi)$ with real profile $R(\varphi)$ in range 16-50 UPR

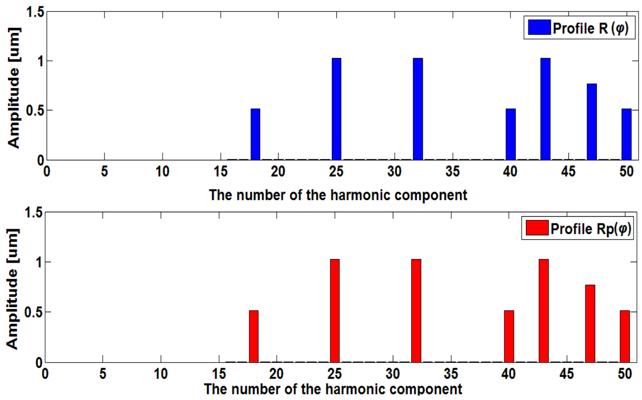

Fig. 8: The bar chart of harmonic components for the profiles in range 16-50 UPR: upper blue chart – real profile *R(φ)*, lower red chart - reconstructed profile $R_p(\varphi)$

As expected, reconstructed profile $R_p(\varphi)$ is overlap with real profile *R(φ)* in range 16-50 UPR (see Fig. 7 and Fig. 8). In this case similar like for transformation in range 2-50 UPR the compatibility of compared profile is very high. The difference between two profiles was also presented in the diagram in Fig. 9.

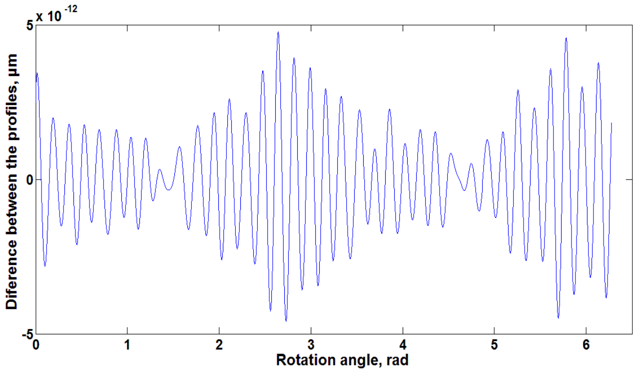

Fig. 9: The difference between the profiles *R(φ)* and $R_p(\varphi)$ in range 15-50 UPR.

The diagram shown in Fig. 9 confirms that the difference between the inspected profiles is negligible. Maximum difference between real profile *R(φ)* and reconstructed profile $R_p(\varphi)$ in range 15-50 UPR equal $4{,}78 \times 10^{-12}$ µm.

The results of computer simulation proved that projection accuracy was very high in the range of 2-50 UPR, as well as for waviness in the range 16-50 UPR.

4. SUMMARY AND CONCLUSIONS

This paper addresses the conception of an adaptation of V-block method known as reference method to waviness measurements of cylindrical surfaces.

The fundaments of the V-block method and characteristic of mathematical model used in this method were presented. This model was developed in order to increase a measurement o V-block method. Additionally computer simulations were carried out to achieve two mains goals. The first is check if proposed mathematical model and V-block method angular parameters are correct. The second goal was verified the concept of the adoption V-block method to waviness measurement of cylindrical elements. The simulation procedures were written in the Mathlab language. Computations were performed with the use of Fast Fourier Transform algorithm. Simulations were carried out in two ranges: The first is 2-50 UPR, which cover roundness and waviness components, and the second is 16-50 UPR, which correspond to surface waviness. In order to visualize the results of the simulations diagrams were plotted that show profiles in Cartesian coordinates and harmonic components of profiles in the form of charts.

Visual comparison of real profile *R(φ)* and reconstructed profile $R_p(\varphi)$ showed that profile obtained by computer transformation is consistent with real profile. The compared profiles difference maximum values is $1{,}41 \times 10^{-11}$ µm in range 2-50 UPR and $4{,}78 \times 10^{-12}$ µm in range 16-50 UPR. In this case we can conclude that the difference between the profiles is insignificant.

Simulations research shows that that parameters of V-block methods and mathematical model are correctly chosen, because projection accuracy of both roundness and waviness is high.

The application of o proper mathematical transformation is necessary to obtain accuracy V-block method to waviness measurements of cylindrical surfaces.

The results of the research work presented in this paper will be a fundamental to develop a measuring instrument that could be used to measure waviness profiles of large cylinders directly on machine tool.

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